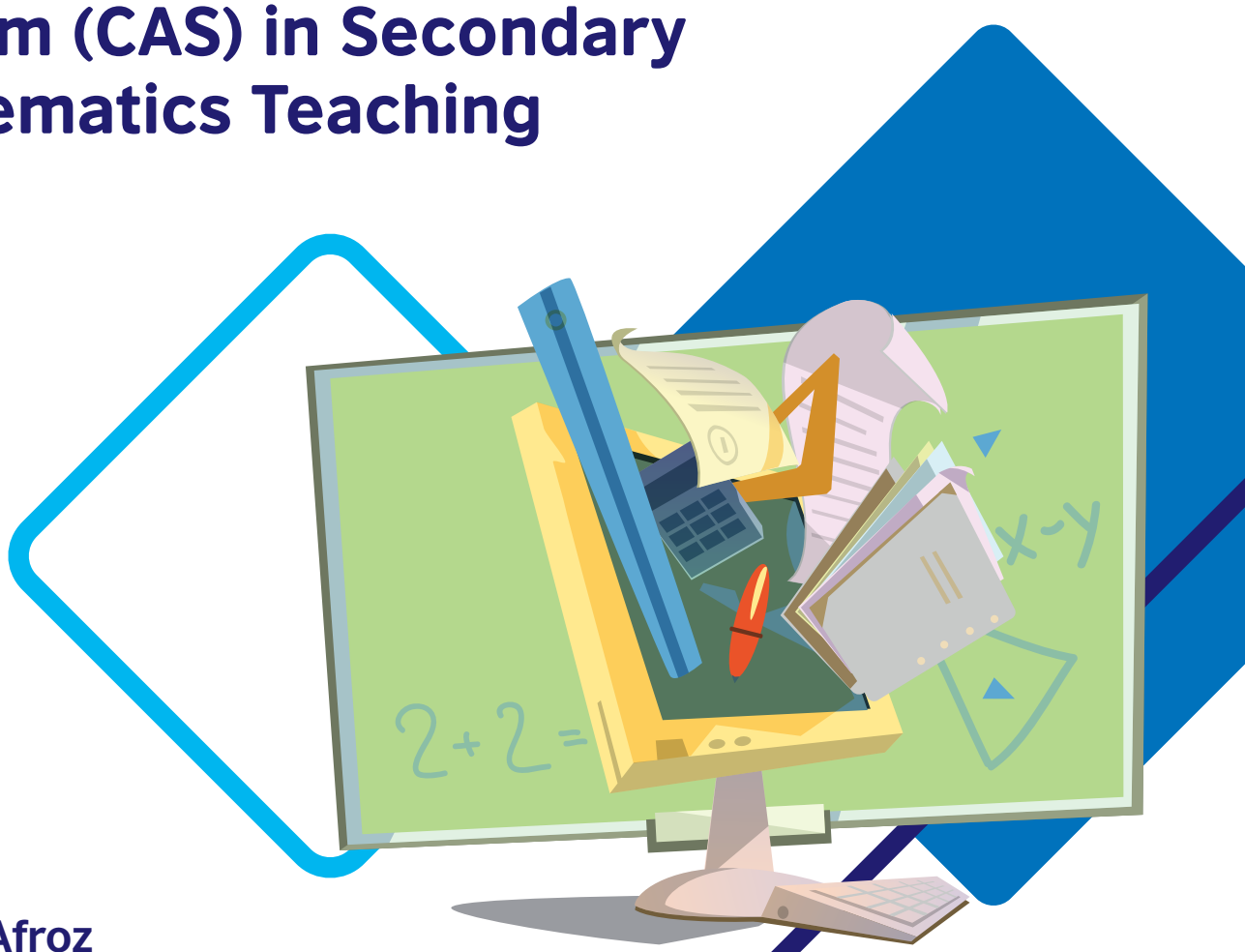
BRITISH COUNCIL

Research

PROJECT REPORT

An Assessment for Implementing Computer Algebra System (CAS) in Secondary Mathematics Teaching



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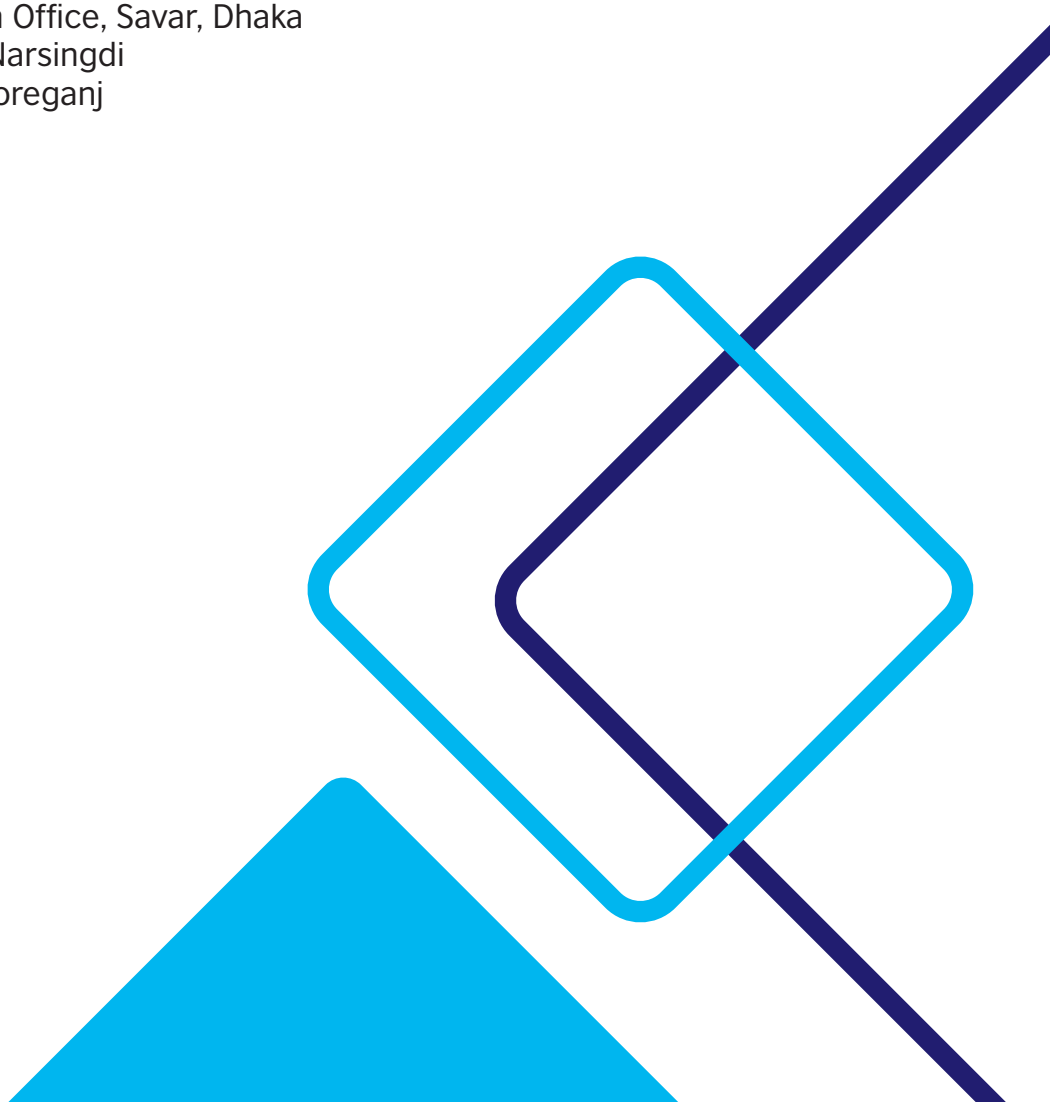


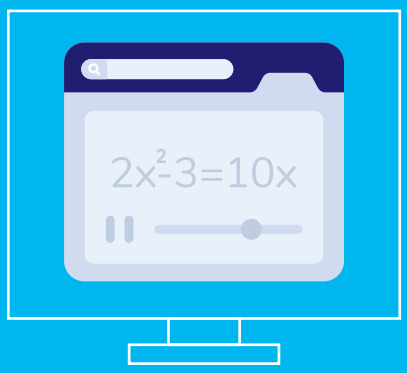
Content

Abstract

This project aims to increase students' interest in mathematics learning through an interactive visual deliberation of mathematical problems. To enhance the teaching and learning methodology for mathematics with the aid of a Computer Algebra System (CAS) at the secondary level, a suitable and effective quality enhancement protocol has to be developed. For this purpose, it is merely important to assess the present status of secondary mathematics education. We perform a sample survey on the present status of the incorporation of ICT in mathematics teaching so that the present scenario of mathematics teaching and learning can be conceived. Based on the survey outcomes, an effective training module is designed for mathematics teaching incorporating CAS to make mathematics teachers competent for dealing with CAS-aided mathematics teaching. As a second step of the research, three training workshops are organized in three different areas to introduce CAS-aided mathematics teaching at the secondary level which could be a milestone to enhance secondary mathematics education. Finally, the project report concludes with research findings and recommendations. It is expected that the outcome of this research will enrich the present research scale for advancing secondary mathematics education.

Keywords: Assessment, Computer Algebra System (CAS), GeoGebra, Mathematics Teaching.

CHAPTER I
INTRODUCTION



"It is security. Certainty. Truth. Beauty. Insight. Structure. Architecture. I see mathematics, the part of human knowledge that I call mathematics, as one thing – one great glorious thing" (Halmos, quoted in Albers, 1986: 127)

Mathematics is perceived as the language of nature that vindicates all other STEM (science, technology, engineering, and mathematics) disciplines, social sciences, business studies, and medical as well. It is undeniable to enhance the mathematical skills and aptitude of a country so that sustainable development goals can be achieved smoothly. Skills of mathematics can facilitate solving different scientific problems in various areas including engineering, computer science, biology, and medicine. Mathematics is the queen of science whereas science depends on mathematics and the world largely depends on science. A famous mathematician Halmos illustrated the meaning of mathematics as follows: *"It is security. Certainty. Truth. Beauty. Insight. Structure. Architecture. I see mathematics, the part of human knowledge that I call mathematics, as one thing – one great glorious thing"* (Halmos, quoted in Albers, 1986: 127) [1].

The aims of quality teaching and learning mathematics are to encourage and enable students to recognise that mathematics permeates the world around us, appreciate the usefulness, power, and beauty of mathematics, and develop patience and persistence when solving problems. Mathematics provides an elective way of building mental discipline and encourages logical reasoning and mental rigor. Moreover, mathematical knowledge plays a crucial role in understanding the contents of other subjects such as science, social studies, and even music and art. Developing investigative skills in Mathematics helps to think precisely, logically, and critically in any given situation. Mathematics has a truly remarkable advancement in the last century from theoretical and applied viewpoints. However, there are numerous studies to highlight the mathematics anxiety of students and teachers from the primary to university level [2].

This is one of the most barriers to developing sufficient mathmatical skills and proficiency. It is also recommended that the use of ICT, e.g. computer algebra system (CAS) can remove the math anxiety among students and teachers significantly. As a result, CAS has been incorporated into the curriculum of secondary mathematics in many countries. Today, it is a challenge to make the CAS familiar among students and teachers for mathematics learning across the globe.

Teaching and learning become a big challenge because education policy has been changed dramatically which move to the digital platform with e-learning nowadays. Mathematics education in Bangladesh has a big threat due to the lack of qualified teachers, a poor foundation of mathematics education, sufficient ICT support in educational institutes, etc. To ensure educational continuity, authorities take challenges to implement different remote options and solutions. Developed country can smoothly take their classes from the beginning of the new curriculum because they have sufficient digital resources and advanced IT platforms. On the contrary, developing country like Bangladesh faces lots of problems for the insufficient IT support and poor digital resources for continuing regular classes with the aid of ICT. Although, an ICT laboratory named *Shiekh Rasel ICT* Lab has been established in almost every secondary institution by the Government of Bangladesh very recently. This is a golden opportunity to utilize such an ICT lab for efective teaching and learning mathematics with the aid of CAS. Moreover, poor internet connectivity in remote areas of the country causes an obstacle to performing efective CAS-aided teaching-learning of mathematics [3].

There are various CAS packages to be used in mathematics teaching some of them are free to use and some of them can be used under a subscription. Among others, GeoGebra is the most suitable and user-friendly CAS package that can be used online and o ine without any subscription. GeoGebra has been popular among students and teachers for the last few decades over the world. In this research, we aim to introduce such a CAS package, GeoGebra to mathematics teachers at the secondary level so that they can be able to introduce it to their students.

During the COVID-19 pandemic [4] all educational institutes of Bangladesh were in lock-down and subsequently, it was decided to conduct the classes in online mode although there were numerous threats to continue online classes, e.g., poor internet connectivity, frequent power failure, unavailability of devices, etc. Education stakeholders e.g. parents, teachers, students, etc. use different types of online platforms to continue their teaching and learning system, such as Google Meet, Facebook, Google Classroom, Zoom, AWWP, etc. Among others, Google meet and Zoom were preferred for meeting and conducting classes in educational institutions, especially secondary institutions. Both students and teachers can communicate with each other smoothly and effectively by using suitable virtual platforms. When the COVID-19 situation was improved, classes were resumed in offine mode in all educational institutions. However, mathematics classes could be more attractive and understandable when a teacher uses a CAS-aided mathematics teaching protocol. As a result, CAS-aided mathematics teaching has also an undeniable role to conduct a blended learning [5] system effectively.

In the mid of 20th century, the necessity of computers was essentially used in many fields, especially in mathematics both for learning and teaching. Nowadays solving mathematical problems by using the DGS (Dynamic Geometry System) and CAS (Computer Algebra System) is strongly recommended for mathematics teachers and students in the modern curriculum, particularly outcome-based education (OBE). GeoGebra package is one of the latest versions of CAS where DGS and CAS are used combinedly as efficient tools for solving mathematical problems.

In mathematics, Cabri is a software for learning geometry mainly at school and an interactive Geometry Software Cinderella which is under the DGS [6], [7], and Mathematica & Maple are under CAS [8] highly deliberated for mathematical problems. GeoGebra, Mathematica, Maple, and Matlab are the leading CAS packages that sustained the users to ensure quality mathematics teaching. The visualization of mathematical problems is in increasing demand as computers play an important role in mathematics teaching since 1990 [9]. Without mathematics, science and engineering problems are unable to be solved whereas CAS has many possibilities for solving a real-life problem [10]. It is argued that “Image or picture can reflect thousands of words” which is truly meaningful for mathematics education. There is a significant demand for visualization in solving mathematical problems. For this purpose, image processing software can be incorporated that reflects thousands of words manually. For the tribute of visualization, critical topics can be developed as simpler topics that are made possible on behalf of the support of CAS. When a student is going to solve a mathematical problem, sometimes it is very difficult to understand how the mathematical problems can be applied in real life. Visualization of a mathematical problem through a CAS package can make it easy to imagine afterward and connect with real life and subsequently the problem could be solved within a very short time although it is not feasible to solve manually. It is strongly commendable that students could understand mathematical problems when the problem is visualized to them with the aid of a CAS package. That is how the CAS package contributes to the visualization of a mathematical problem notably.

Bender and Schreiber [11] elaborately discussed the construction of the graphical conception of an algorithm which is meaningful for the collaborations about the structure of the algebraic equations. It is possible to recreate the graphical constructions at any time with any parameters or variables involved in a mathematical equation where the students can see the transformations, translations, shifting, and reflections of a graph from the given functions by using the CAS package, e.g., GeoGebra which is well known as the most user-friendly graphing utility across the globe.

Mathematics education at the secondary school level with the aid of CAS is as follows

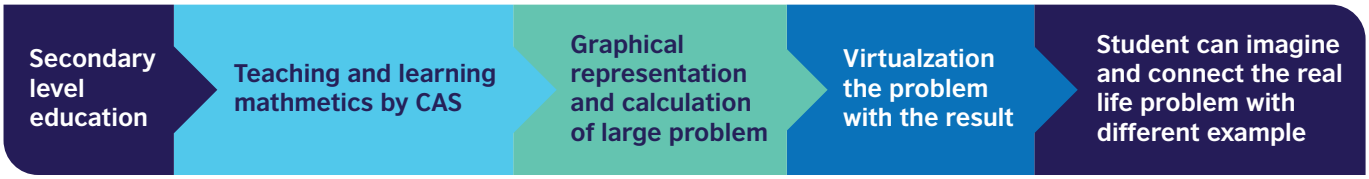


Figure 1: Framework with CAS at secondary Mathematics.

The research project aims to address the following issues:

- To conceive the present status of the teachers' knowledge, skill, and impression regarding the use of ICT (CAS) in mathematics teaching through a sample survey.
- To organize three training workshops at three different places in the country to introduce CAS-aided mathematics teaching and learning at the secondary level.
- To reveal the limitations and threats to incorporating the CAS package in the curriculum of mathematics at the secondary level.

Under the aforementioned goals of the project, a survey on CAS-aided mathematics teaching has been performed where more than 200 mathematics teachers participated from different secondary institutions in rural and urban areas. Based on the survey outcomes, a training module has been developed to conduct the training workshop on the use of CAS in mathematics teaching. Most of the participants appreciated the initiative and appealed to arrange such type of workshop regularly. It was expressed that they gathered a brief introduction about CAS-aided mathematics teaching through the training workshop and what they could apply in the classroom in making mathematics teaching and learning more attractive and enjoyable. The outcome of the sample survey and feedback from the training workshops are presented here in scientific form. The rest of the project report is organized in the following manner.

In section 2, an overview of mathematics education at the secondary level has been presented including the learning objectives of mathematics, curriculum, syllabus of secondary mathematics, and learning outcomes on completion of the mathematics course, etc. Furthermore, the availability of ICT support in the secondary school and teachers' qualifications & training have also been pointed out in section 2. We have discussed the use of Computer Algebra System (CAS), e.g. GeoGebra, Mathematica, Maple, Maxima, Octave, and Matlab, etc. for mathematics teaching in section 3. Section 4 represents the scenario of CAS-aided mathematics teaching during COVID-19 with a few examples for the implementation of the CAS package, GeoGebra. We have exhibited the outcome of the survey on CAS-aided mathematics teaching performed in different rural and urban areas in section 5. Section 6 displays the participants of the training workshop and their feedback rigorously which will be supportive in developing a sustainable CAS-aided mathematics curriculum. Finally, the project report concludes with section 7 which includes the findings of the research project along with recommendations to implement a CAS-aided mathematics curriculum successfully.

CHAPTER 2
Present Status of Mathematics Education at Secondary Level

In this study, we described the current state of mathematics education at the secondary level in Bangladesh. We conceive a great variety of research approaches for different areas such as rural areas, villages, and city areas.

2.1 Learning Objectives of Secondary Mathematics

Mathematics is an important part of human thought & logic and essential to understand ing real-life problems

- To improve understanding of some of the mathematical concepts and logic, which are important in secondary school mathematics students for increasing their mathematical thinking and understanding.
- To increase the ability to specify subject matter involved in a specific mathematics topic and make distinctions among them. To improve understanding of various teaching strategies, compare and use different assessment techniques.
- To develop mathematics skills after secondary school level and participation at a pre-university level which is the mission and vision of secondary education.
- To use mathematics and pedagogy knowledge flexibly in authentic situations through field experiences with secondary students under the supervision of highly qualified, experienced teachers and university supervisors.

Students should expect to:

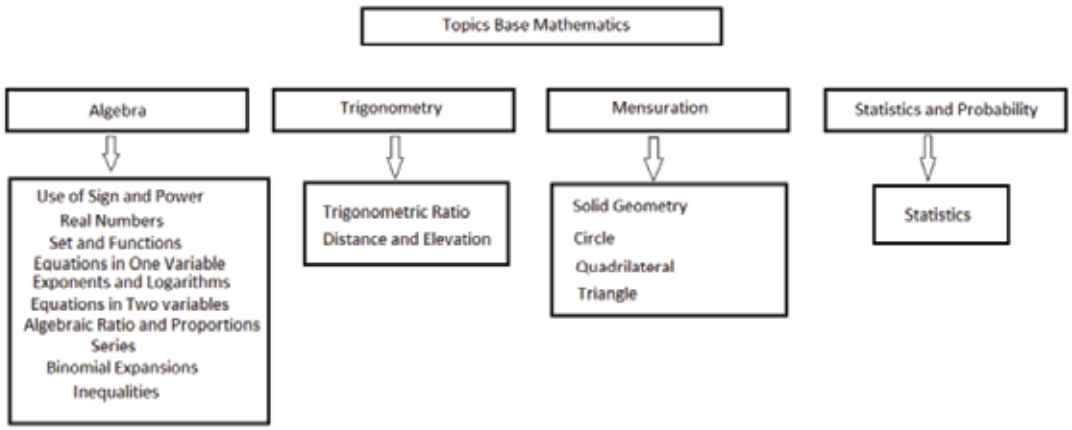
- learn new things about secondary school mathematics
- after completing each topic try to solve the exercises with a real-life example
- uses communication software (e.g., word processors, e-mail, PowerPoint) not as objects of our discussion but in simple ways to generate and share products, assignments, and ideas
- learn something about each of several common types of mathematics software
- learn how to make decisions about how to use technology as an effective mathematics teacher

There are different types of teaching methods that can be categorized into four broad types as follows

1. Teacher-centred methods
2. Learner-centred methods
3. Content-focused methods
4. Interactive/participative methods.

2.2 Syllabus of Secondary Mathematics

The syllabus of secondary mathematics aims to enable the students to acquire mathe matical concepts and develop skills for continuous learning from the secondary level to the pre-university level. To develop thinking skills, communication with the textbook to real-life problems, and metacognitive skills through a mathematical approach to solve the problems. To make the student enthusiastic in the field of mathematics to develop the processes, metacognition and attitudes are embedded in the learning experiences. Number and Algebra, Geometry and Measurement, and Statistics & Probability are the three components of the syllabus of secondary-level mathematics.
Secondary school mathematics can be categorized into four sections Algebra, Trigonometry, Mensuration, and a part of Statistics and Probability.



There are mathematics courses at the secondary level of Bangladesh, one is “**General Mathematics**” is compulsory for all students and the is “**Higher Mathematics**” which is an elective subject for the science group students.

2.2.1 General Mathematics (Compulsory)

1. Set and Function
2. Algebraic Expression
3. Exponents and Logarithms
4. Practical Geometry
5. Circle
6. Trigonometric Ratios
7. Finite Series
8. Mensuration
9. Statistics

2.2.2 Higher Mathematics (Elective)

1. Algebraic Expressions
2. Geometry
3. Summation of Infinite Series
4. Trigonometry
5. Exponential and Logarithmic Functions
6. Binomial Expansion
7. Coordinate Geometry
8. Probability

2.3 Mathematics Curriculum

Subject: **Mathematics** (Compulsory)
Text Book: Secondary Mathematics (Class-IX & X)
By *National Curriculum and Textbook Board (NCTB),* Bangladesh
(Bengali and English Version)
Marks Distribution: 100 (Theoretical: 100)

Question Pattern & Marks Distribution				
Question Type	Group	Part	Remarks	Marks
C.Q.	Group A Group B Group C Group D	Algebra Geometry Trigonometry Statistics	Any 7 questions have to be answered	10 X 7 = 70
M.C.Q.	Full Syllabus			1 X 30 = 30
Total (C.Q. + M.C.Q.)				100

Subject: **Higher Mathematics** (Elective)
Text Book: Higher Mathematics (Class-IX & X)
By *National Curriculum and Textbook Board (NCTB)*, Bangladesh
(Bengali and English Version)
Marks Distribution: 100 (Theoretical: 75, Practical: 25)

Question Pattern & Marks Distribution				
Question Type	Group	Remarks	Instruction	Marks
C.Q.	Group A: Algebra Group B: Geometry Group C: Trigonometry & Probability	2 questions 2 questions 2 questions	Any 7 questions have to be answered	10 X 5 = 50
M.C.Q.	Full Syllabus			1 X 25 = 25
Total (C.Q. + M.C.Q.)				75
Practical				25
Total				100

2.4 Learning Outcomes

After successful completion of the mathematics course, students will be able to deal with a mathematical problem efficiently [12]

• After completing real numbers, students gather knowledge on how to classify dif ferent types of numbers, and carry out addition, subtraction, multiplication, and division, to solve different relevant problems.

• Sets and functions gives the idea of sets, subsets, infinite sets, finite sets, union and intersection sets, power sets, ordered pairs, Venn diagrams, determine the domain and range, to draw diagrams for functions respectively.

• Algebraic expressions are important to expand formulae of square and cube, remain der theorem, factoring from algebraic formulae for solving real-life problems, and solving the problems by applying formulae.

• Students will learn to explain the rational exponent, apply the positive integral exponents, solve problems, explain logarithms, and explain natural and common logarithms from Exponents and Logarithms.

• At the end of the chapter Equations in One Variable, the student will be able to explain the concept of variable, the difference between equation and identity, solve linear equations, form linear equations based on real-life problems, solve the quadratic equations and find the solution sets, from the quadratic equations based on real-life problems and solve.

• Describe the basic postulates of plane geometry, prove the theorems related to tri angles, and apply the theorems and corollaries related to triangles to solve problems by the topic Lines, Angles, and Triangles.

• Practical Geometry explains triangles and quadrilaterals with the help of figures, construct triangles by using given data, construct quadrilateral, parallelogram, and trapezium by using given data.

• Circles explain arcs, angle at the center, angle in the circle, quadrilaterals inscribed in the circle, prove theorem related to circle, apply of the theorems to solve many problems related to a circle, state constructions related to circle.

• Trigonometric ratios of acute angles, determine the mutual relations among the trigonometric ratios of an acute angle, solve and prove the mathematical problems justifying the trigonometric ratios of an acute angle, determine and apply trigonometric ratios of acute angles 300, 450, 600 geometrically, meaningful, trigonometric ratios 00 and 900, prove and apply the trigonometric identities.

• Distance and Elevation explain the geoline, vertical plane, and angles of elevation and declination, solve a mathematical problem related to distance and height with the help of trigonometry, and measure practically different types of distances and heights with the help of trigonometry.

• Algebraic Ration and Proportion explain algebraic ratio and proportion, use different types of rules to transformation related to proportion, learn successive propor tion, use ratio, proportion, successive proportion in solving real-life problems.

• Simple Simultaneous equation in two variables to verify the consistency of simple simultaneous equations related to real-life mathematical problems, solve the simul taneous equations with two variables by graphs.

• Finite series describe the sequence and series and determine the differences between them, explain finite series, from formulae for determining the fixed term of the series and the sum of the fixed numbers of terms and solve mathematical problems by ap plying the formulae. Determine the sum of squares and cubes of natural numbers, solve mathematical problems by applying different formulae of series, construct for mulae to find the fixed term of a geometric progression and sum of fixed numbers of terms, and solve mathematical problems by applying the formulae.

• Ratio, Similarity, and symmetry explain geometric ratios, the internal division of a line segment, verify and prove theorem related to ratios and similarity, concepts of symmetry, verify line and rotational symmetry of real objects practically.

• Area-related theorems and constructions explain the area of polygons, verify and prove theorems-related areas, construct polygons and justify construction by using given data, a quadrilateral with area equal to the area of a triangle, construct a triangle with area equal to the area of a quadrilateral.

• Mensuration determines the area of polygon region by applying the laws of an area of triangle and quadrilateral and solving allied problems, determining the circumference of the circle and the length of the chord of a circle, area of a circle and its segment (solve the allied problems), the area of uniform and non-uniform solids.

• Statistics explain cumulative frequency, frequency polygon, and ogive curve, data by the frequency polygon, and ogive curve, the method of measuring of central tendency, the necessity of short-cut method in the measurement of central tendency, find the mean, median, and mode, explain the diagram of frequency, polygon and ogive curve.

2.5 ICT Support at Secondary School

Information and communication technology, or ICT as it is more often known, has emerged as an effective tool for advancing educational quality on a global scale. It was introduced into education with the hope that it would make teaching and learning interesting for the students. ICTs can be creatively used anytime, anyplace to transform a typical teacher-centered classroom into a learner-centric one and provide the access to information through telecommunication [13]. In education areas, ICT is performing a vital role in teaching and learning, especially in developing countries such as Bangladesh, Nepal, Sri Lanka, etc. ICT helps the school teachers for attractive performance in the classroom which ensures quality teaching.

When Bangladesh's national ICT strategy was created in 2009, it saw ICT as a tool for the country's overall development [14]. ICT was to be a useful tool in the policy's intended curriculum, pedagogy, and teacher capacity building improvements. This includes educating teachers and students at the primary, secondary, and university levels about ICT literacy. Therefore, paying close attention to these factors is essential to get the most out of using ICT in secondary school. The government of Bangladesh is using ICT into systems for management and information, monitoring and mentoring, and ongoing professional development. ICT includes the use of computers, the internet, and electronic communication [15]. In Bangladesh, ICTs are playing an important role in the learning process, offering powerful tools for expanding educational access and improving skills and knowledge.

According to UNESCO "ICT is a scientific, technological and engineering discipline and management technique used in handling information, its application and association with social, economic and cultural matters". For the improvement of the student's quality, a skilled teacher is the first requirement. ICT helps both students and teachers in learning and teaching which can make dynamic changes in the education sector. Nowadays, the accessibility and affordability of ICT are increased exponentially not only in urban areas but also in remote areas.

2.6 Teachers' Qualifications and Training

The development of a country is depending on the development of the educational sector where skilled teachers can play a vital role. Mathematics school teachers need some training for their better performance in the classroom. Teaching Mathematics is a course of the B.Ed. program (a training of secondary school teachers) that provides a range of learning opportunities designed specifically for trainees seeking to become secondary mathematics teachers. The purpose of this course is to support trainees to develop the knowledge, skills, and attitudes required to competently teach mathematics in the secondary school curriculum for years 6-8 and years 9-10 of Bangladesh.

To ensure the quality education of mathematics school teachers need training on their subject, administrative capacity, and some professional training. There are four teen teacher training colleges in Bangladesh under the National University and running the following programs:

- Bachelor of Education (B.Ed.)
- Master of Education (M.Ed.)

A trained teacher can be able to improve their teaching quality with the following steps

- Teacher conduct their class with full of confidence.
- Strict to follow accurate class time.
- Teachers should make their teaching materials with much care.
- Try to make the topic interesting and involve the students more attentively.
- Teacher should use appropriate and updated teaching methods.
- Follow the recent-most perfect techniques of lecture presentation.
- Try to confirm the perfect learner's assessment etc.

2.7 Obstacles of Mathematics Education at Secondary Level

According to each student's unique demands, teachers will provide them worksheets or assignments that have been properly produced. For math teachers, the biggest diffculty is to lessen students' "Mathematics Phobia." Math is typically thought of being a dull, abstract topic. However, it can be challenging to teach every topic using examples from the real world, such as the concepts in algebra. Many students believe that having little to no mathematics background is adequate to pursue many jobs. They are unaware of its numerous uses [16, 17, 18]. The IT sector is playing a vital role in conducting the classes and lively interactions of the students with education. As a result, the education stakeholders continue their activities by using Zoom, Google Meet, Google Classroom, Module, AWWP, etc. But the teachers are not proficient to work with the above-mentioned ICT tools. It is a big challenge to make mathematics interesting, especially in the case of virtual classes. Numerous studies have been performed in investigating the challenges of mathematics teaching at the secondary level in different aspect, e.g. scarcity of classrooms, effective teaching materials, ICT supports, and qualified teachers, etc.

Implementation of such ICT tools could make mathematics more interesting and enthusiastic remarkably. The following obstacles are frequently observed to conduct ICT aided mathematics teaching

- Frequent power failure
- High price of Internet package
- Lack of adequate training to conduct CAS-aided mathematics teaching
- Scarcity of devices, e.g., laptop, tab, smartphone, etc.

CHAPTER 3
Computer Algebra System (CAS)

Nowadays, without teachers' and students' cognitive and practical ideas about mathe matics, it's very difficult to continue during this pandemic with the online class which is possible with CAS. During off-line classes, CAS has a vital role to make understandable and make the imagination connect mathematics and the applications of mathematics to real-life problems. Several CAS packages can facilitate mathematics teaching at the secondary level, such as

* **GeoGebra**
* **Maple**
* **Mathematica**
* **Maxima**
* **Octave**
* **MatLab**

GeoGebra is one of the most convenient and free CAS packages and the most user friendly mathematics teaching tool. As a result, we choose the GeoGebra CAS package to introduce it among the mathematics teachers at the secondary level in Bangladesh.

3.1 GeoGebra: A CAS Package

GeoGebra (a portmanteau of geometry and algebra) is an interactive geometry, algebra, statistics, and calculus application, intended for learning and teaching mathematics and science from primary school to the university level. GeoGebra is available on multiple platforms, with apps for desktops (Windows, macOS, and Linux), tablets (Android, iPad, and Windows), and the web. The official webpage of GeoGebra is https:**//www.geogebra.org/** where GeoGebra can be used on an online platform and its app can also be downloaded.

GeoGebra was created by Markus Hohenwarter in 2001/2002 as part of his master's thesis in mathematics education and computer science at the University of Salzburg in Austria. For the pre-university level and university level, mathematics education can be demonstrated and visualized by the GeoGebra software which can give us a wide space in different topics. GeoGebra [7], [8] aimed to encourage students to approach mathematics experimentally such as by drawing the graphs (show the intersection points of two conic sections perpetuate widely in Cabri and it can show how an ellipse becomes a hyper bola), solve algebraic equations, differentiation, integration. At a pre-university level, it is interesting for investigating the change of a circle's equation by changing parameters, directly changing the circle, change the equation of the line. The base of GeoGebra is projective and Euclidean geometry in the real plane where secondary school students can also expand and simplify the equations, and draw graphs of algebraic equations. GeoGebra has powerful tools were the commands for differentiation, and integration finding the slope in a straight line. GeoGebra has all the abilities which are demanded from a suitable drawing/designing software [6] for the student's and teachers' learning outcomes in mathematics [7], [8].
GeoGebra is software with wide coverage due to its different representations. GeoGebra for demonstration and visualization.

GeoGebra is a new software system that integrates possibilities of both dynamic geometry and computer algebra in one tool for mathematics education. GeoGebra is also recognized as a construction tool in Mathematics. GeoGebra and discovering mathematics, GeoGebra for preparing teaching materials [9]. GeoGebra has all the abilities which are demanded from a suitable drawing/designing software [6]. GeoGebra is based on projec tive and Euclidian geometry in the real plane. Equations are expanded and simplified symbolically and a special grammar for arithmetical expressions in school notation has been implemented. In most dynamic geometry systems, users will encounter jumping objects in certain constructions. For example, the intersection points of two conic sections may permutate wildly in Cabri when an ellipse becomes a hyperbola. For the differentiation and integration of functions in one variable the open-source computer algebra system JSCL is built into GeoGebra. GeoGebra is an interactive geometry software [8],[11] that also offers algebraic possibilities like entering equations directly. It is aimed at students (aged 10 to 18) and teachers in secondary schools. The program encourages students to approach mathematics experimentally. For example, it is possible to investigate the parameters of a circle's equation by dragging the circle with the mouse. On the other hand, students may also manipulate the equation directly and see the changed circle in the geometry window. These constructions may be altered dynamically by dragging free objects. Furthermore, it is possible to enter the coordinates of points or vectors, equations of lines, conic sections or functions, and numbers or angles directly. Hence, from the very beginning, the software has been designed for use in schools.

3.2 Interface of GeoGebra with examples

GeoGebra is one the most versatile tools for mathematics education in secondary schools. The treatment of problems should not be affected by system-caused translations. Manipulations ought to be possible in a familiar way [19]. Big efforts have been made to allow input in school notation:

For example a line g may be entered as g: 3x + 4y = 7 or a circle c as c: $(x-3)^2+(y+2)^2=25$. Also, calculations with geometric objects like points and vectors are feasible. The centroid of a triangle with vertices A, B and C might be entered as S = (A + B + C)/3. Additionally, GeoGebra offers many powerful commands starting from the slope of a straight line up to differentiation and integration of functions. Finally, to teach mathematics effectively, it might be used in many different ways e.g., for demonstration and visualization.

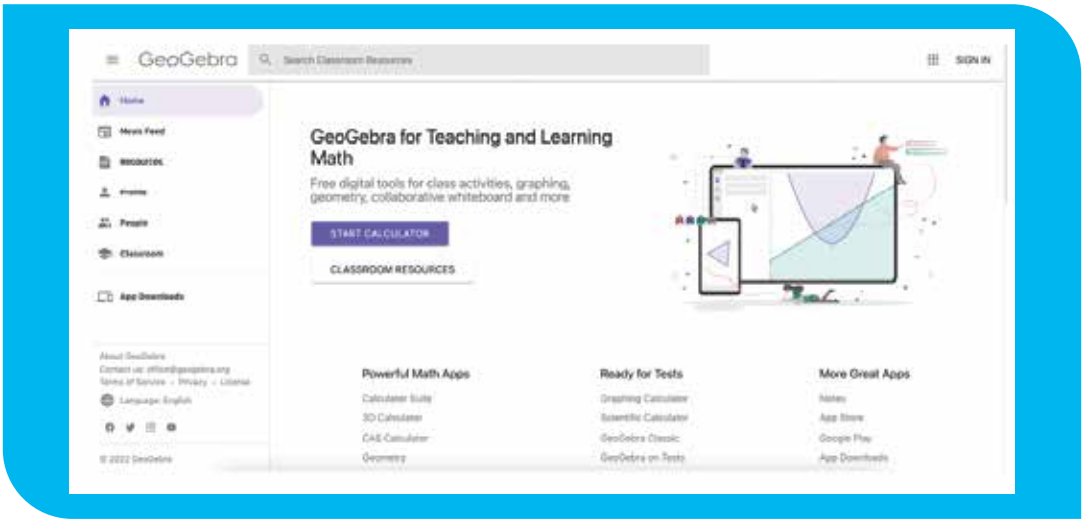


Figure 2: Interface of the GeoGebra webpage

Figure 3: Interface of the GeoGebra calculator.

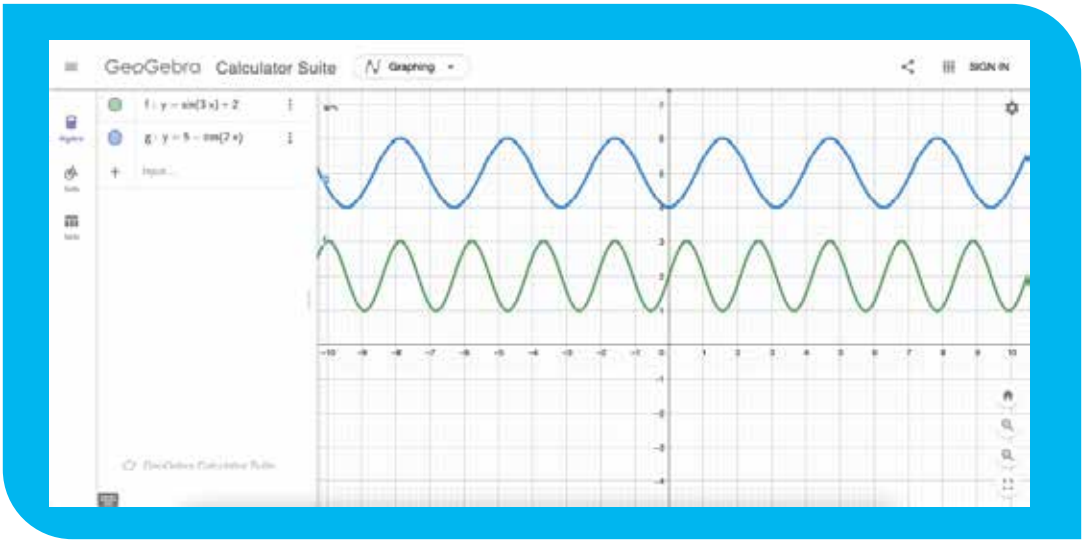


Figure 4: Examples in the GeoGebra interface.

CHAPTER 4
CAS-aided Mathematics Teaching during Covid-1

During COVID-19, when online classes were performed, it was deeply realized that a CAS package has to be imposed in mathematics teaching across the globe. For solving a problem and visualization equations, CAS has a pivotal role. The use of CAS packages,e.g., GeoGebra could make the virtual math classes more interactive and interesting.

4.1 Use of CAS at the Secondary Mathematics

To create a strong fascination with sciences, it is extremely important to make mathematics interesting and understandable to secondary students because this is the entry point for a student to be attracted to the science discipline. That is why classroom teaching and learning of mathematics have to be made enjoyable and interactive for the students. We strongly believe that such an effective interaction would be possible through CASaided mathematics teaching [21], [22]. Each and every topic of the syllabus of secondary mathematics can be demonstrated precisely using GeoGebra so that the students gather a clear concept of elementary mathematics and that will make them inquisitive for further investigations.

4.2 Examples of CAS-aided Mathematics

We demonstrate a few examples of mathematical problems using the CAS package, GeoGebra

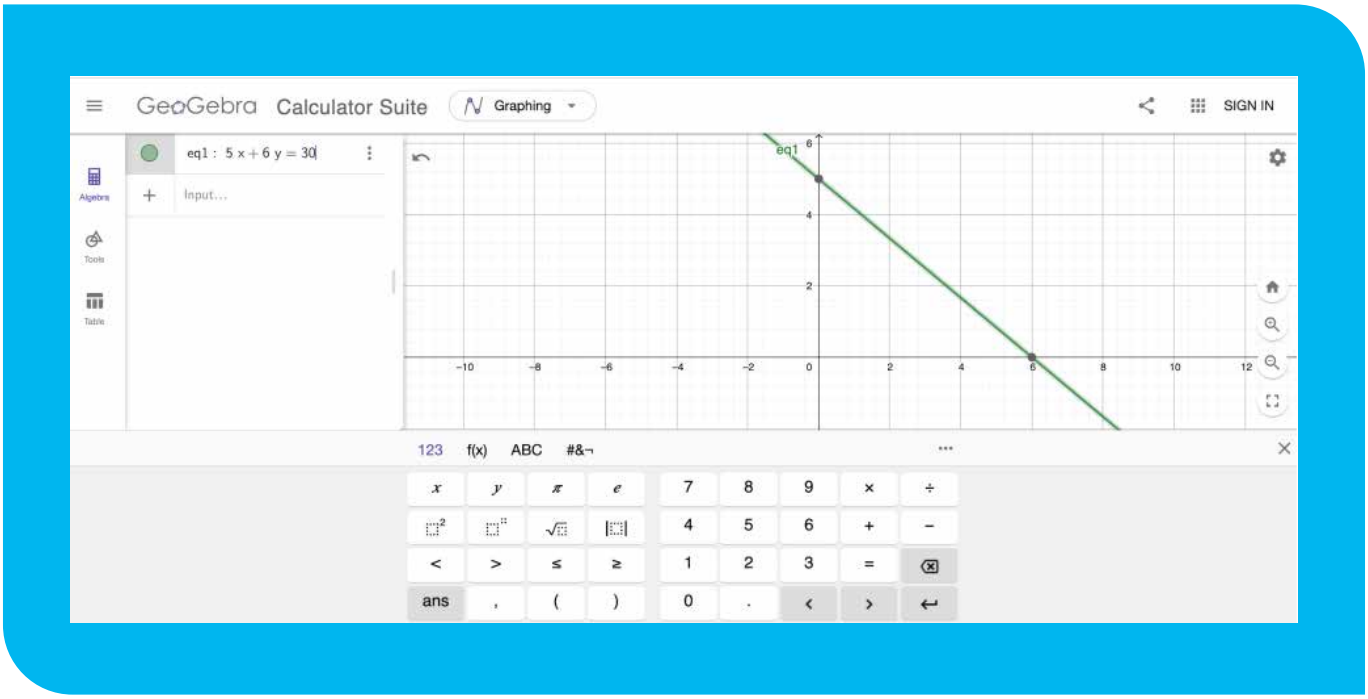


Figure 5: Graph of 5x + 6y = 30

We consider an equation (Fig. 5) of a straight line 5x + 6y = 30. GeoGebra shows a quick result for the graph of the equation under the graphing calculator

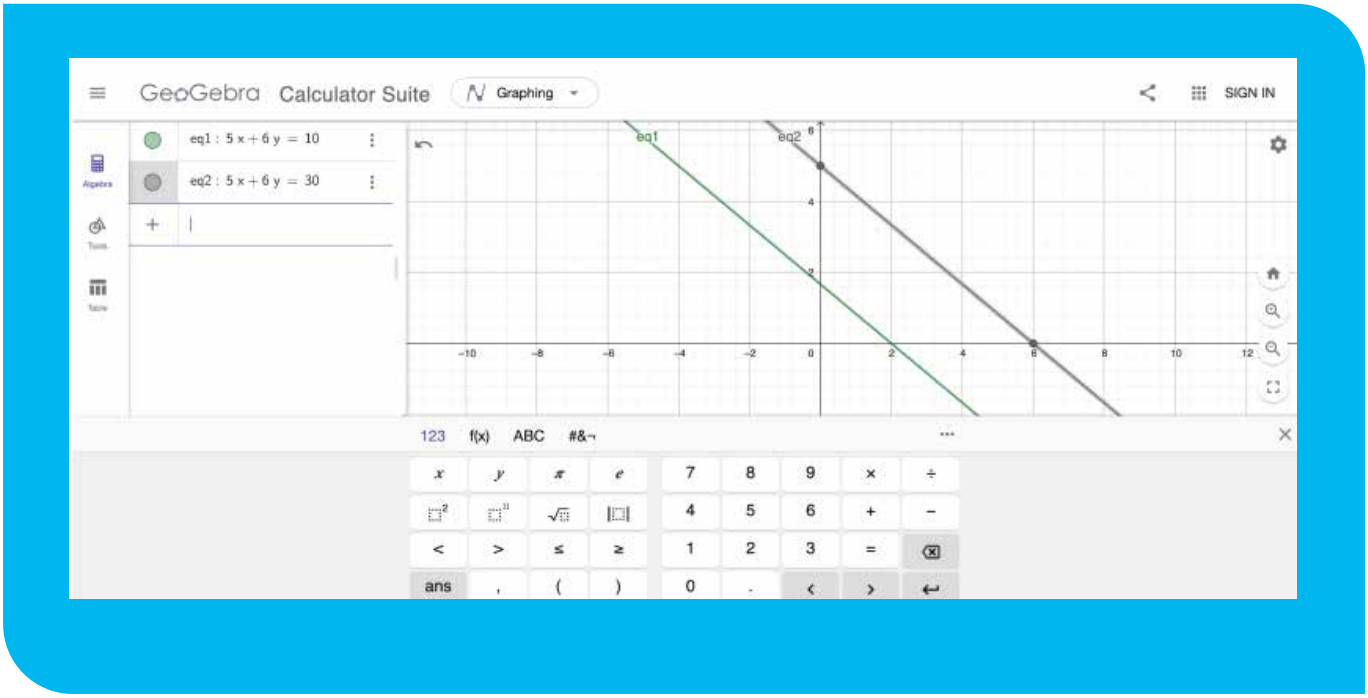


Figure 6: Graph of 5x + 6y = 10 & 5x + 6y = 30

As a second example, we include one more equation to the previous one and check their graphs using GeoGebra. It is observed that these two equations exhibit two parallel straight lines (Fig. 6) as expected.

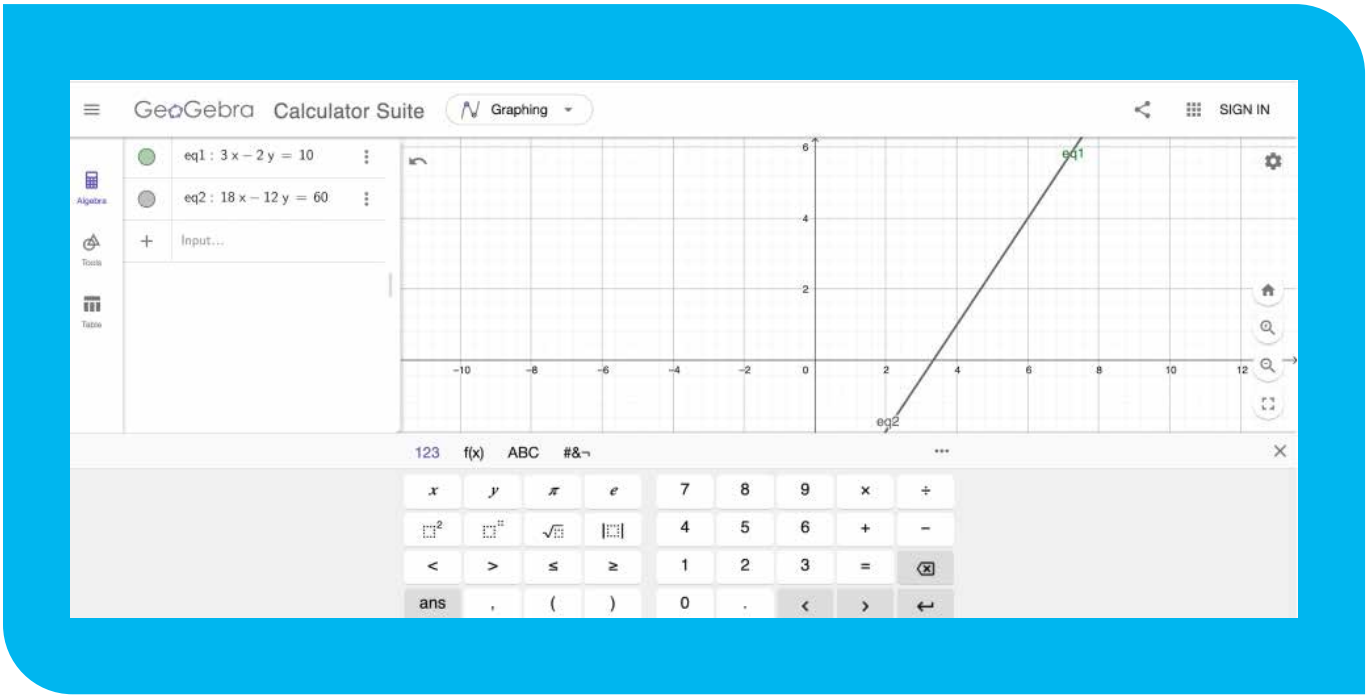


Figure 7: Graph of 3x - 2y = 10 & 18x - 12y = 60

We consider two equations (Fig. 7) of straight lines: the first equation is 3x - 2y = 10 and the second equation is 18x - 12y = 60. We draw these equations on the same coordinate axes and observe that these two straight lines overlap each other.

Figure 8: Graph of 2x + y = 10 & x + 12y = 20

Again, if we draw another two equations (Fig. 8) of straight lines on the same coordi nate axes and the equations are 2x + y = 10 and x + 12y = 20 equation of the straight line by using GeoGebra. We find that these two straight lines intersect at a point which is the solution of these two equations. GeoGebra can show such type of examples which can make the problem understandable to the students. Students' thinking capacity could be increased and they can imagine the visualization of the math equations.

CHAPTER 5
Sample Survey and Analysis

It is extremely important to conceive the present scenario of mathematics teaching in corporating ICT in Bangladesh. For this purpose, a sample survey was performed under this research project which plays a pivotal role in the project. A survey on ICT-aided Mathematics teaching was conducted comprising a questionnaire (Appendix- A) where more than 200 mathematics teachers participated in the survey from di erent corners of the country. In order to perform a survey, simple random sampling has been performed considering four di erent regions: rural area, suburban area, urban area, and metropoli tan area. The sample size of 200 has been chosen for the survey in the following manner: rural area - 70, suburban - 70, urban - 40, and metropolitan - 20. One of the major goals of this research is to assess the present status and prospects of ICT-aided mathematics teaching in rural and suburban areas as there are not adequate ICT support for conducting effective mathematics teaching. Although the sample size chosen is relatively smaller due to the inadequate time of the project period. However, the data was collected con sidering the rural area, suburban area, urban area, metropolitan area, etc. which justifies the rationale of the sample.

5.1 Survey on ICT-aided Mathematics Teaching

The survey on "Teaching and Learning Mathematics using Computer Algebra System (CAS) at Secondary Level" was performed based on the following questionnaire:

(Q1) Do you think the use of ICT is important to teach mathematics at the secondary level?

(Q2) Are you currently using any computer algebra system (CAS) package to teach mathematics?

(Q3) Are you having any ICT-related problems teaching mathematics at the secondary level?

(Q4) Do you have any training in ICT-aided mathematics teaching?

(Q5) Are you interested to attend any training workshop on ICT-aided mathematics teaching?

All the participants in the survey fully agreed with Question 1 that the use of ICT is important to teach mathematics at the secondary level. It is evident from the survey that only 2.33% of math teachers are using computer algebra systems to teach mathematics at the secondary level. 93% mathematics teachers face problems to use ICT in teaching mathematics. It is also found that only 9.3% of mathematics teachers have elementarylevel training in ICT-aided mathematics teaching. All the participants in the survey are willing to attend any workshop on ICT-aided mathematics teaching. The responses to the survey questionnaire are presented in a compact satisfied form in Figures 4-8.

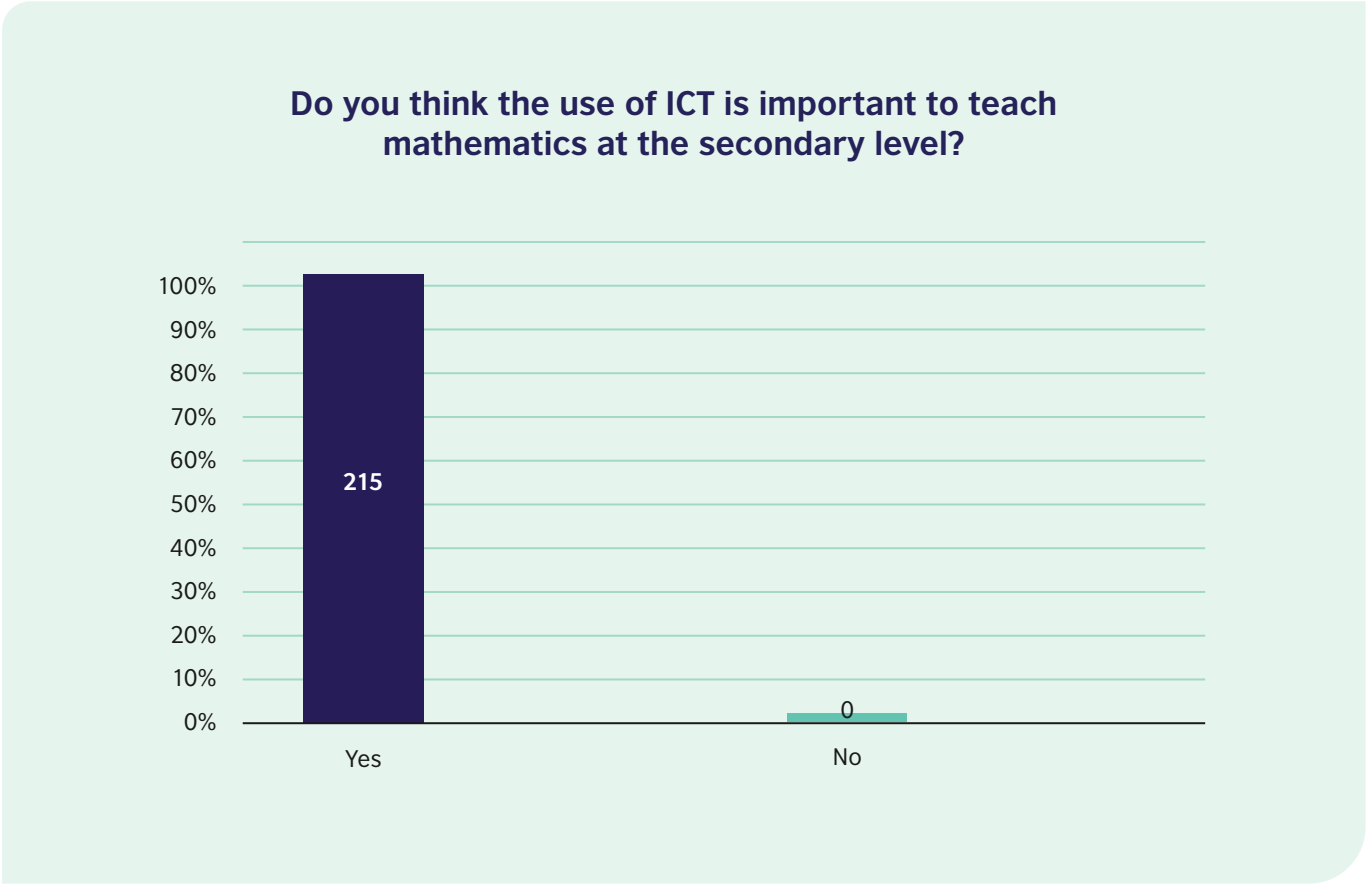


Figure 9: Response to the Question 1.

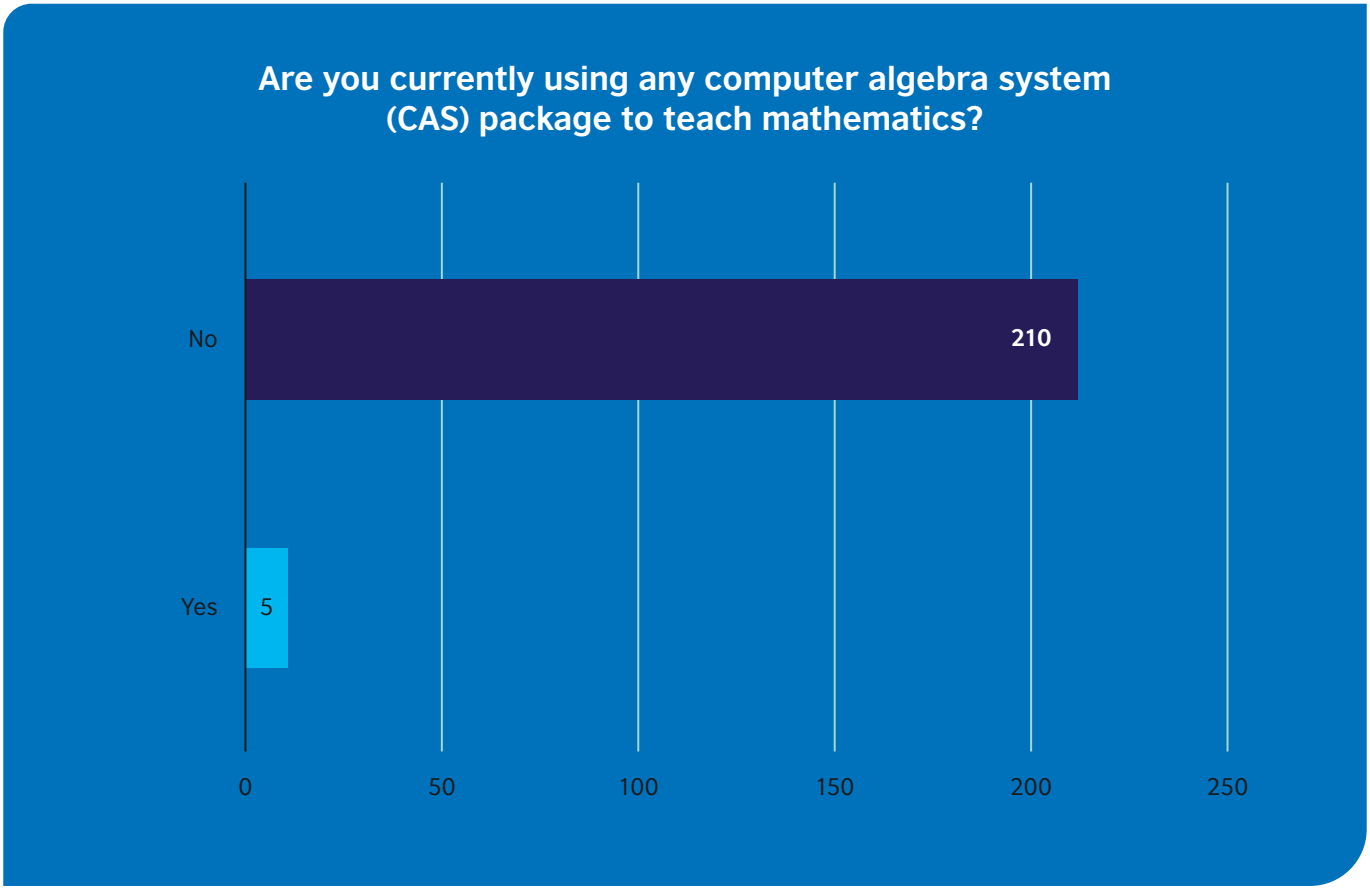


Figure 10: Response to the Question 2.

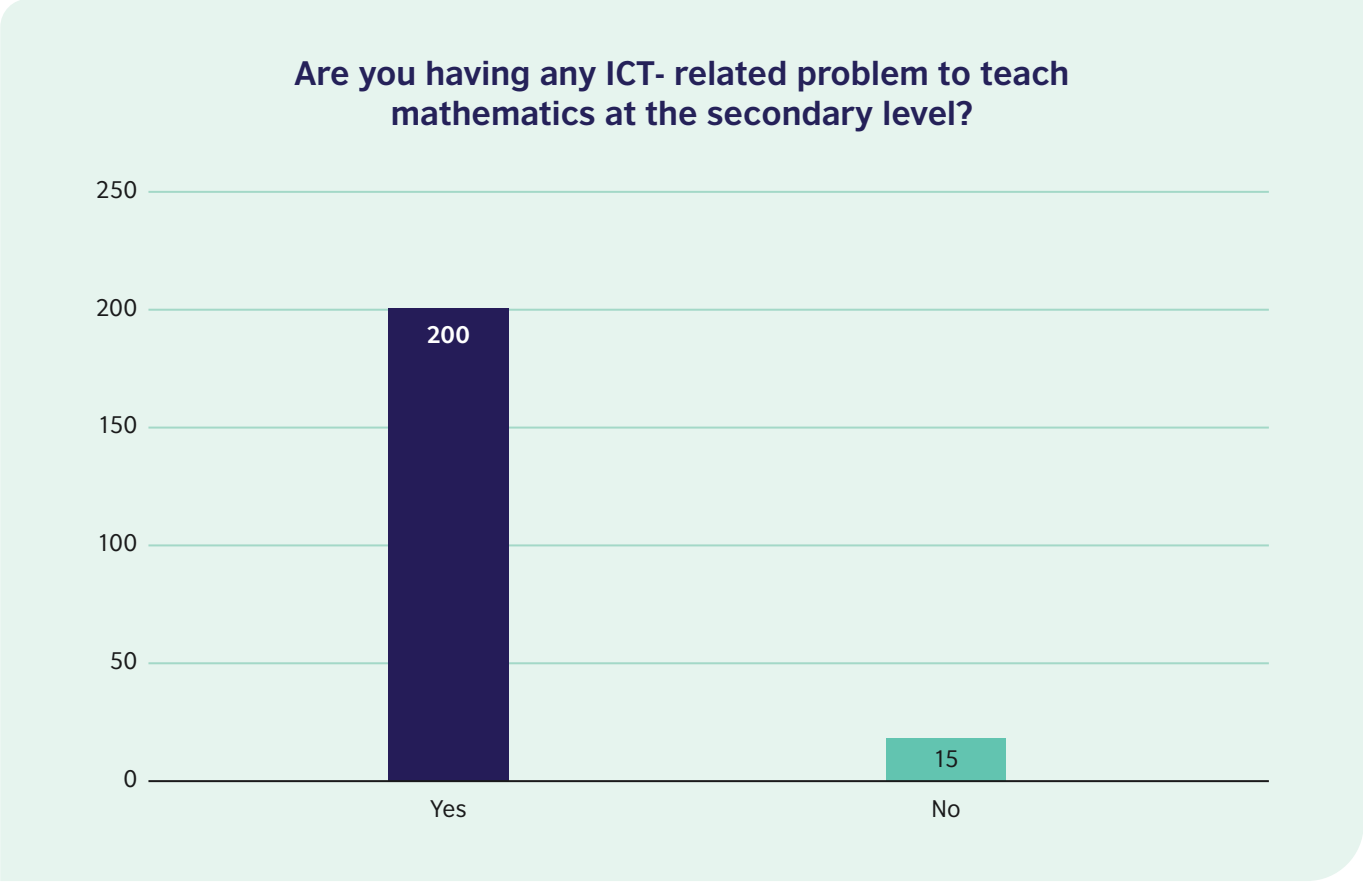


Figure 11: Response to the Question 3.

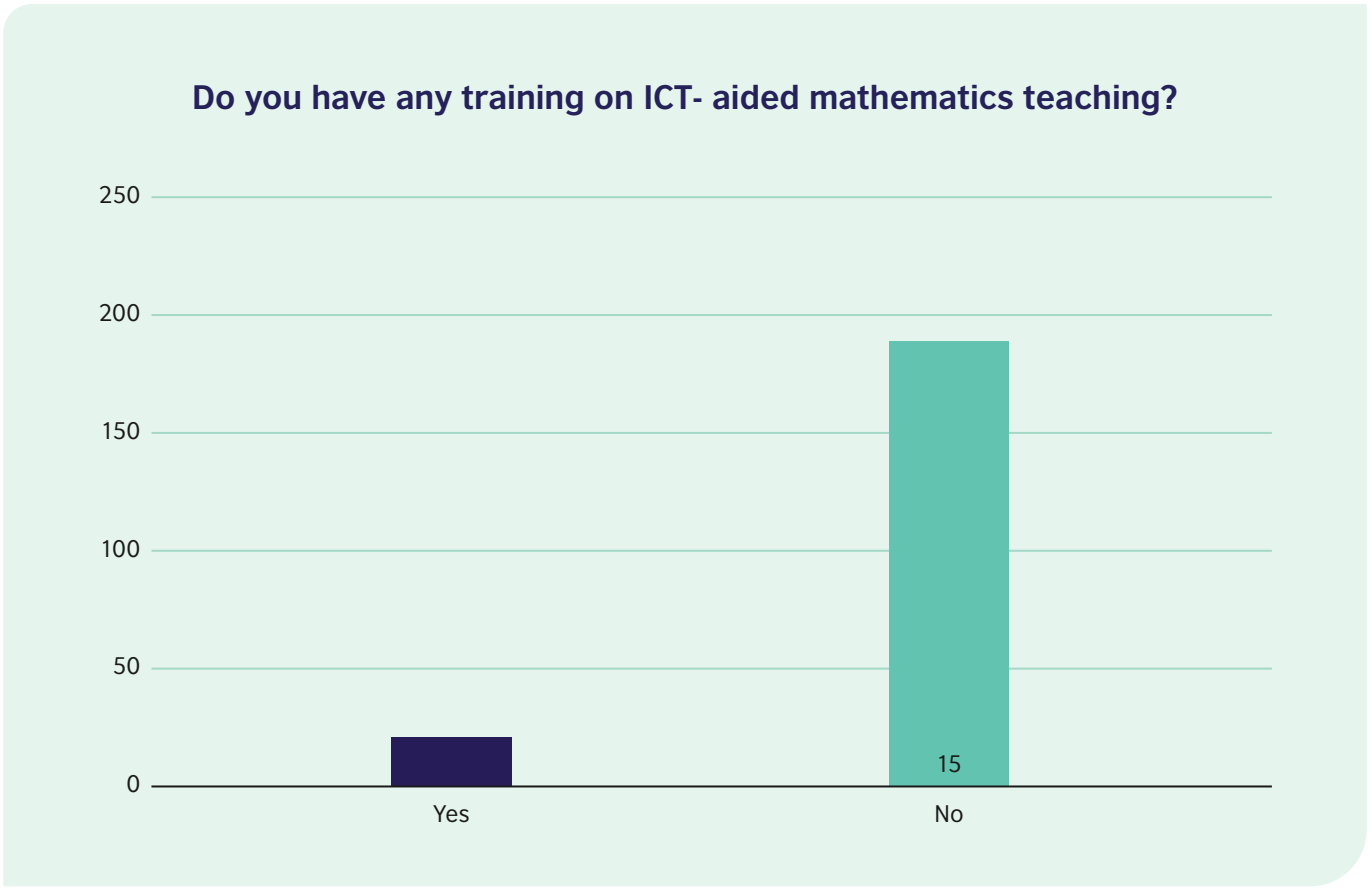


Figure 12: Response to the Question 4.

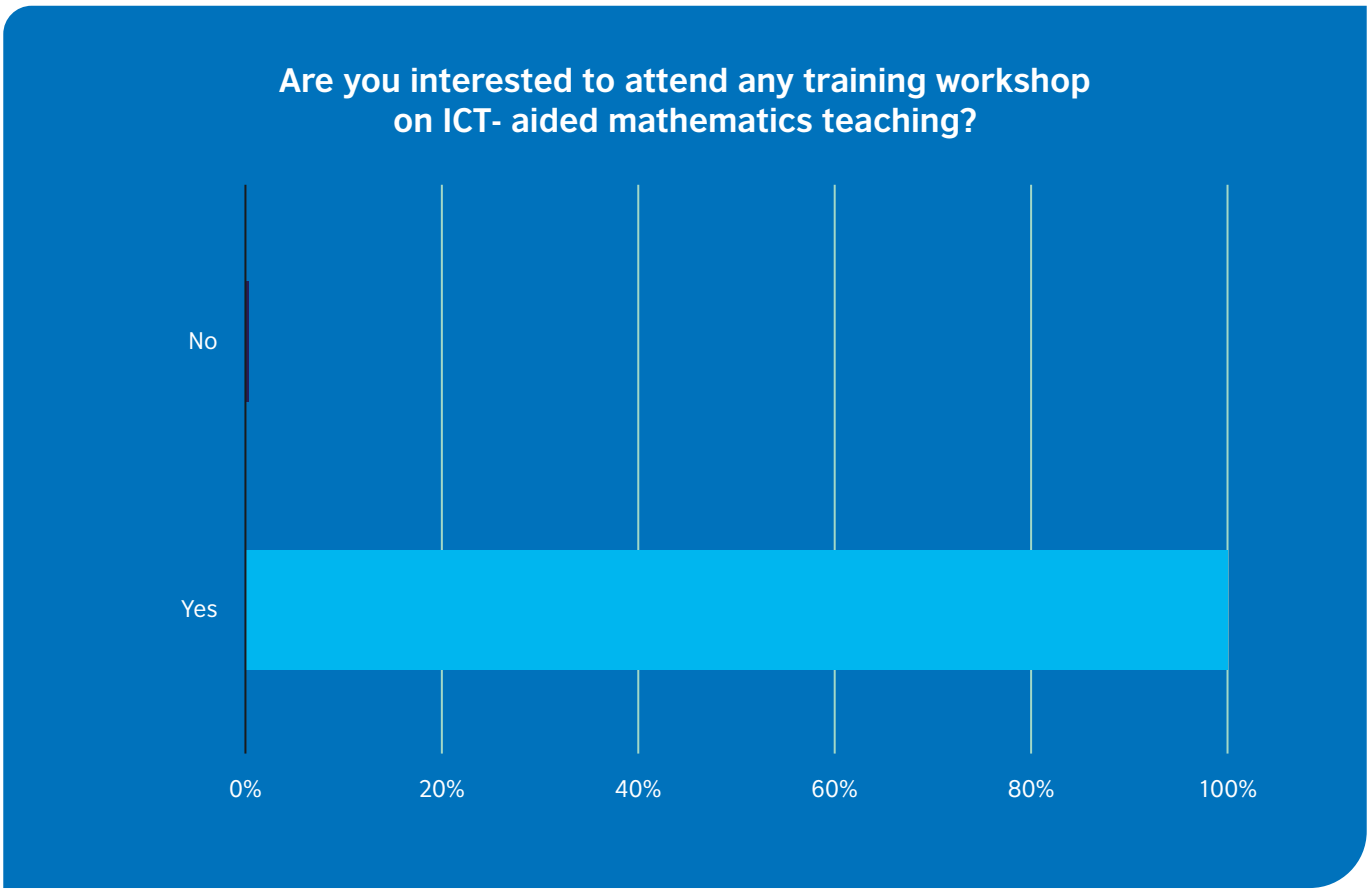


Figure 13: Response to the Question 5.

CHAPTER 6
Training Workshops on CAS-aided Mathematics Teaching

Based on the sample survey, a training module for secondary mathematics teachers was developed by respective experts. We arranged three training workshops at three different places under the research project to introduce CAS-aided mathematics teaching. First training workshop was organized in the Abdul Aziz High School, Kishoreganj (rural area), 2nd one was organized in the Narsingdi BIAM Zila School, Narsingdi (Suburban area), and in the Department of Mathematics, Jahangirnagar University (urban area).

6.1 Training Workshop at the Narsingdi District

A training workshop on "Teaching and Learning Mathematics using Computer Algebra System (CAS) at Secondary Level" was organized on 21 May 2022 at the Narsingdi BIAM Zila School, Narsingdi.39 mathematics teachers from different secondary institutions participated in the workshop and passed their feedback.

6.1.1 Participants of the Workshop

The participants of the workshop are listed in the following table:

Name	Qualification	School
Mahbubur Rahman	M.Sc.	Balusahar High School
Sukumar Chandra Kar	BMC, B.Ed.	Panchdona Sir K.G.G. High School
Abu Bakr Shamim Ahmed	BSc, BEd	Baduarchar Kazi Abul Hashem High School
Yamin Mia	B.Sc., B.Ed.	Ghoradia High School
Jannatul Ferdessi	B.Sc , M.Sc , B.Ed.	Dighirpar Dakhil Madrasa
Meherunnesa	B.Sc., B.Ed.	Madhabadi Sati Prasanna Institution
Md. Mozammel Haque	M.Sc.	Atirpara Kalikumar Institution School
Md. Anisur Rahman	B.Sc , M.Sc	Gazairchar Asiya Islamia Alim Madrasa
Sharmin Akhter Rima	B.Sc., M.Sc.	Palash Thana High School
Md. Zakir Hossain	BSc, B.Ed.	Madhabadi Girls School and College
Md. Didar Hossain	BSc, B.Ed.	Hajipur Basiruddin Memorial High School
Md. Mojibur Rahman	BSc, B.Ed, MS	Narsingdi Govt. Girls High School
Hasan Saifullah	B.Sc. , M.Sc.	Maningson Int. School and College
Md. Shafiqul Islam	B.Sc., B.Ed.	Bonaid Babu Mia High School
Md. Lutfar Rahman	B.Sc., B.Ed.	Manohardi Govt. Pilot Model High School
Tipu Mollah	B.Sc., M.Sc.	Ekduaria High School
Md. Adil Hossain	M.Sc.	Raban High School
Md. Chan Mia	B.Sc, B.Ed.	Pubali Jute Mills High School
Tapas Chandra Debnath	B.Sc., M.Sc., B.Ed.	Kazi Mofiz Uddin School
Md. Hazrat Ali	B.Sc., M.Sc.	Narsingdi Pilot High School
Mizanur Rahman Jewel	M.Sc.	Paulanpur Islamia Fazil Madrasa
Md. Zahirul Islam	B.Sc., M.Sc.	Narsingdi Biam District Schoo
Md. Abul Kasham	B.Sc., B.Ed.	Sabuj Pahar High School
Md. Tajul Islam	BSc, B.Ed.	Duttergaon High School
Md. Abul Halim	B.Sc., M.Sc.	Nasima Qadir Mollah High School & College
Md. Fazlur Rahman	BSc, B.Ed.	Chalakchar High School
Md. Safiul Haque	BSc, B.Ed.	Jamekasemia Kamil Madrasa
Md. Kamal Uddin Sarkar	BSc, B.Ed.	Mutiullah Bhuiyan High School

Name	Qualification	School
Md. Amanur Hossain	B.Sc., M.Sc.	Charsindur Girls High School
Md. Moazzem Hossain	BSc, B.Ed.	Charmadhabpur Madinatul Uloom Madrasa
Ranushukla Das	BSc, B.Ed.	Bhuiyan Islamia Dakhil Madrasa
Swapan Chandra Debnath	M.Sc., B.Ed.	Noadia Khaishakhali Girls High School
Abdur Rauf	M.Sc.	Goaldi Sakiatunnesa Dakhil Madrasa
Md. Sajeeb	Diploma in Eng.	Karimpur K. K Boys High School
Md. Zahirul Islam	M.Sc., B.Sc.	Dulalpur High School
Md. Iftekharul Haque	B.Sc., B.Ed.	Brakshandi Girls High School
Md. Amjad Hossain	B.Sc., B.Ed.	Kanthalia Union High School
Md. Aliullah	M.Sc.	Chandbari Islamia Kalim Madrasa

Academic qualifications of the participants in the workshop can be figured in the following manner: 57% of the participants have the qualification B.Sc., 7% of the participants have B.Sc. with B.Ed., 29% have M.Sc. degree, and 7% have M.Sc. degree with B.Ed. degree as presented in the following diagram.

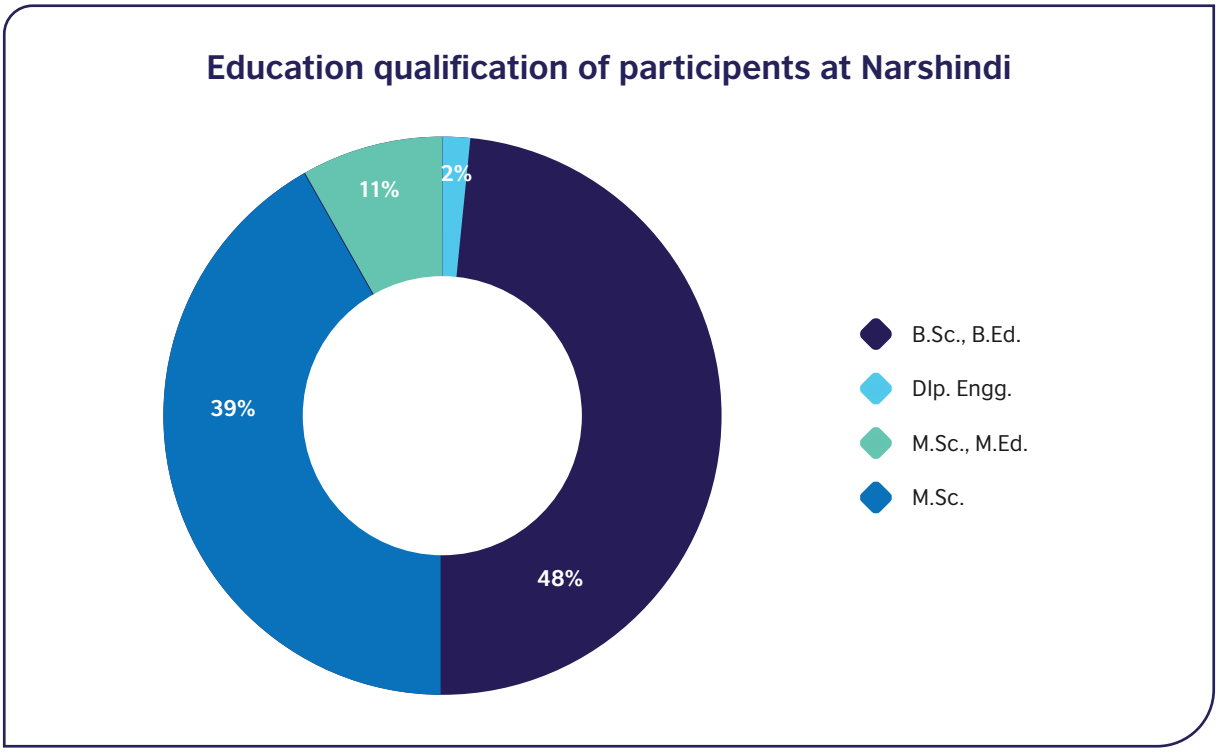


Figure 14: Academic Degree of the Participants in the Workshop.

6.1.2 Feedback from Participants

At the end of the workshop, a feedback form was filled out by all the participants of the workshop that consists of the following questionnaire:

Q1. Recently, reluctance towards mathematics education is being observed among the students at the secondary level.

Q2. The use of ICT is important to ensure quality mathematics teaching at the secondary level.

Q3. Computer Algebra System (CAS) will play a significant role in alleviating students' math fears and anxiety.

Q4. The use of ICT can facilitate graph sketching at the secondary level of mathematics.

Q5. I have been informed about the role of the CAS-aided mathematics teaching through this workshop.

Q6. Training workshops on the role of the CAS-aided mathematics teaching should be continued.

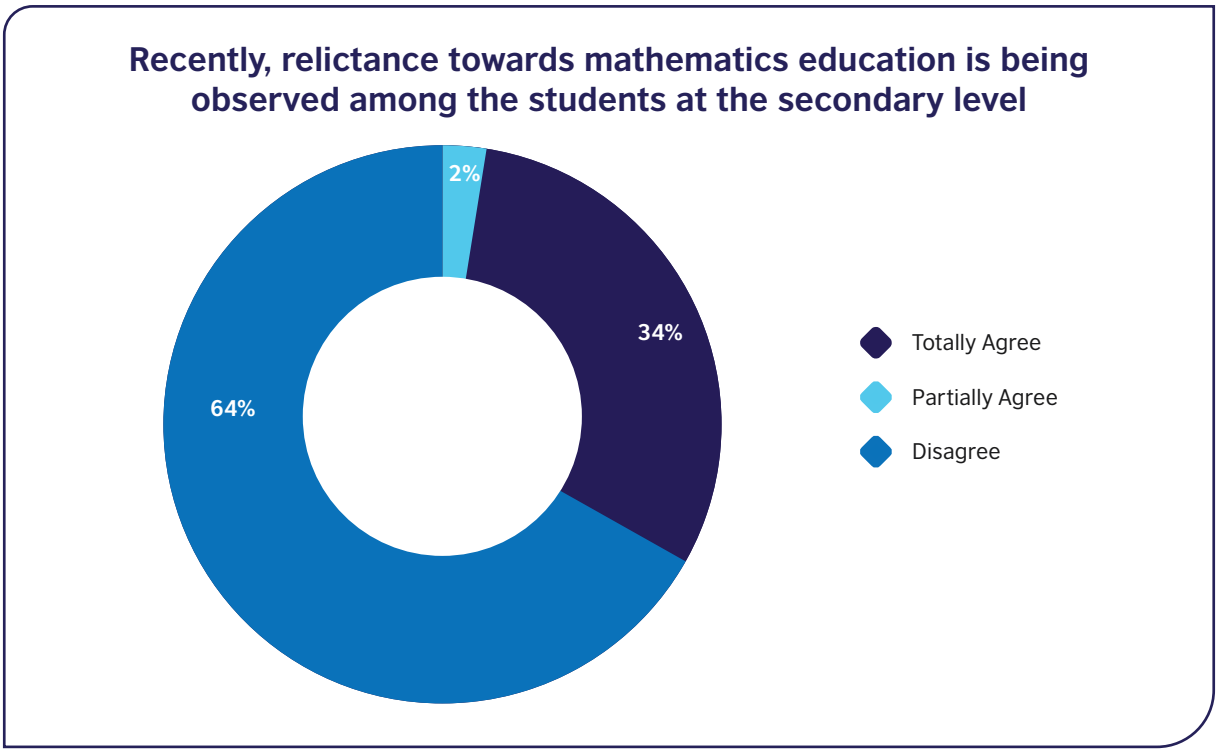


Figure 15: Response to the Question 1.

It is observed that 64% of the participants totally agreed with (Q1) while 34% agreed partially, and 2% disagreed with the question.

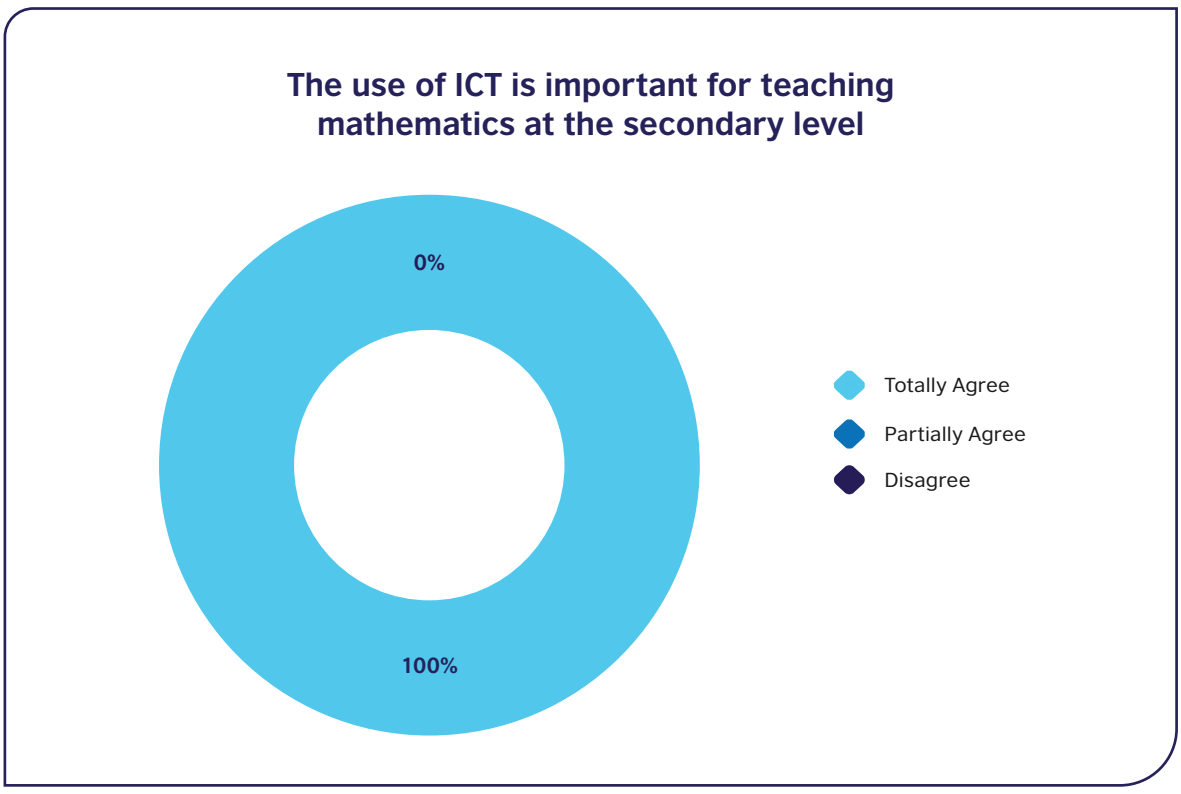


Figure 16: Response to the Question 2.

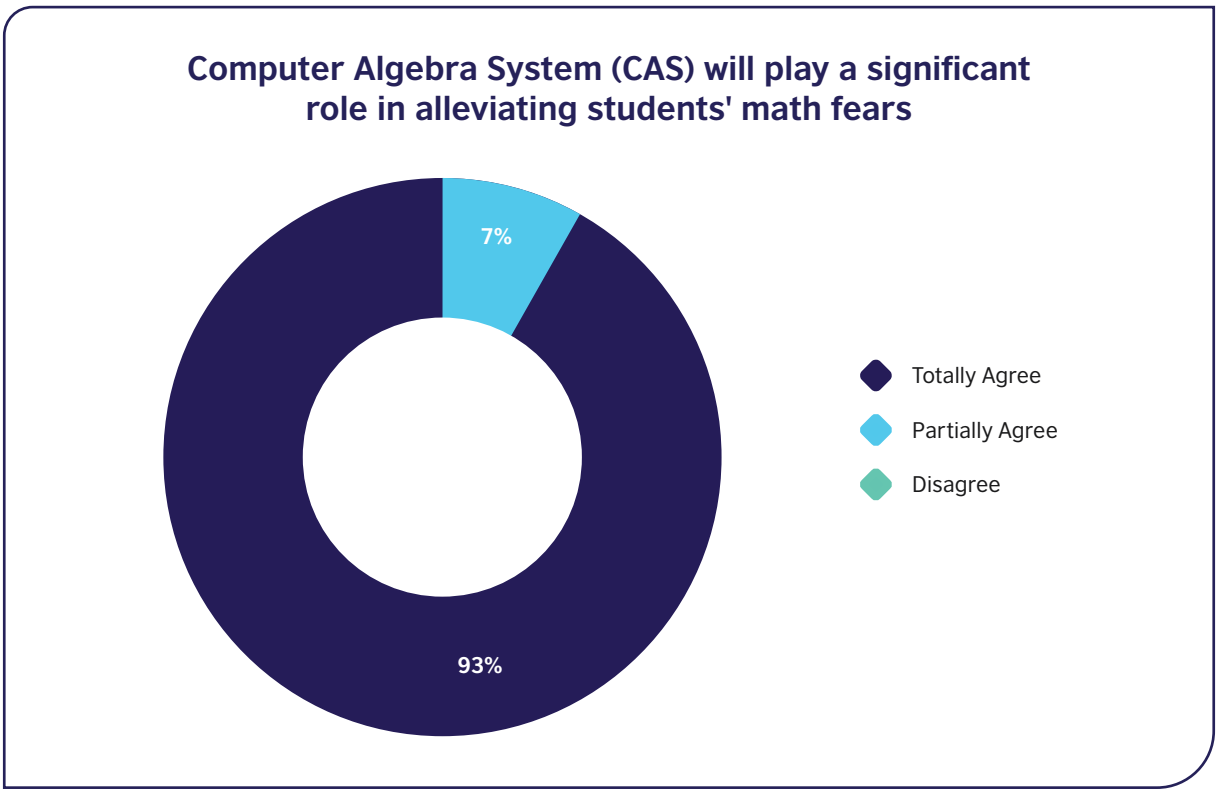


Figure 17: Response to the Question 3.

Question 2 was totally agreed upon by 100% of the participants. Most of the partici pants (93%) agreed totally with question 3 while 7% agreed partially.

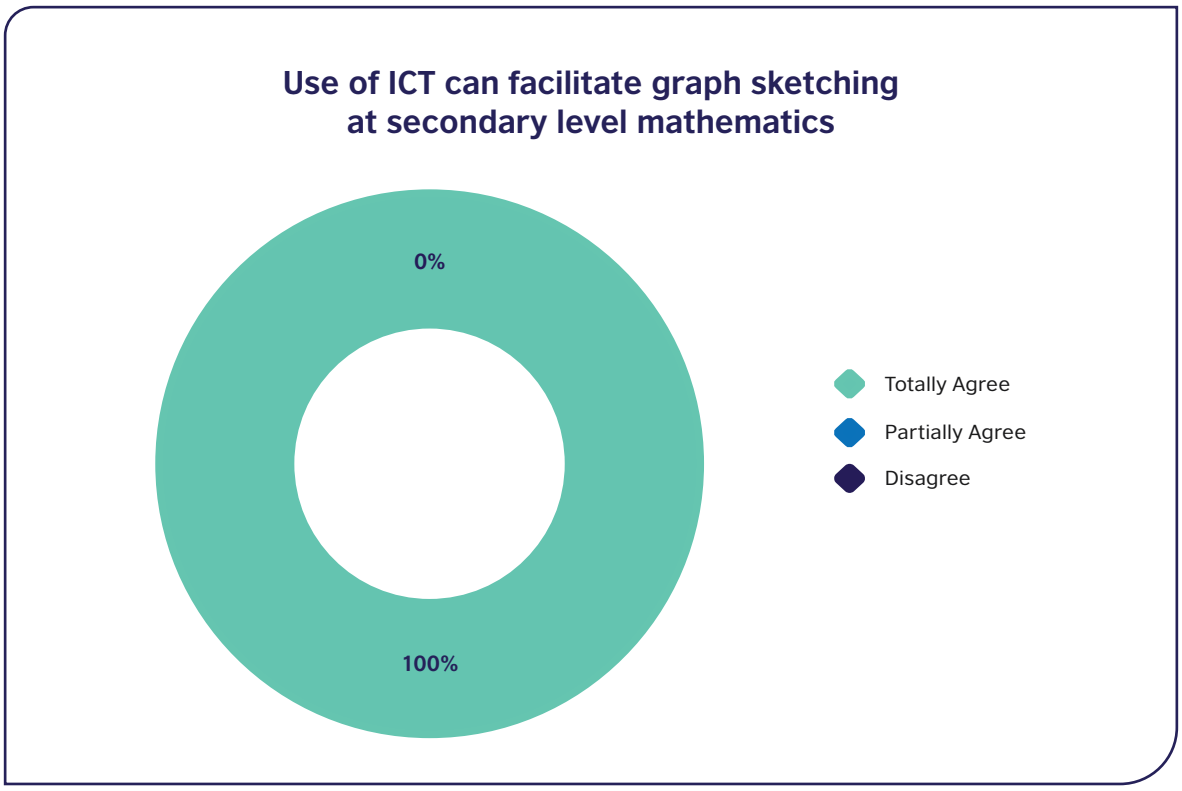


Figure 18: Response to the Question 4.

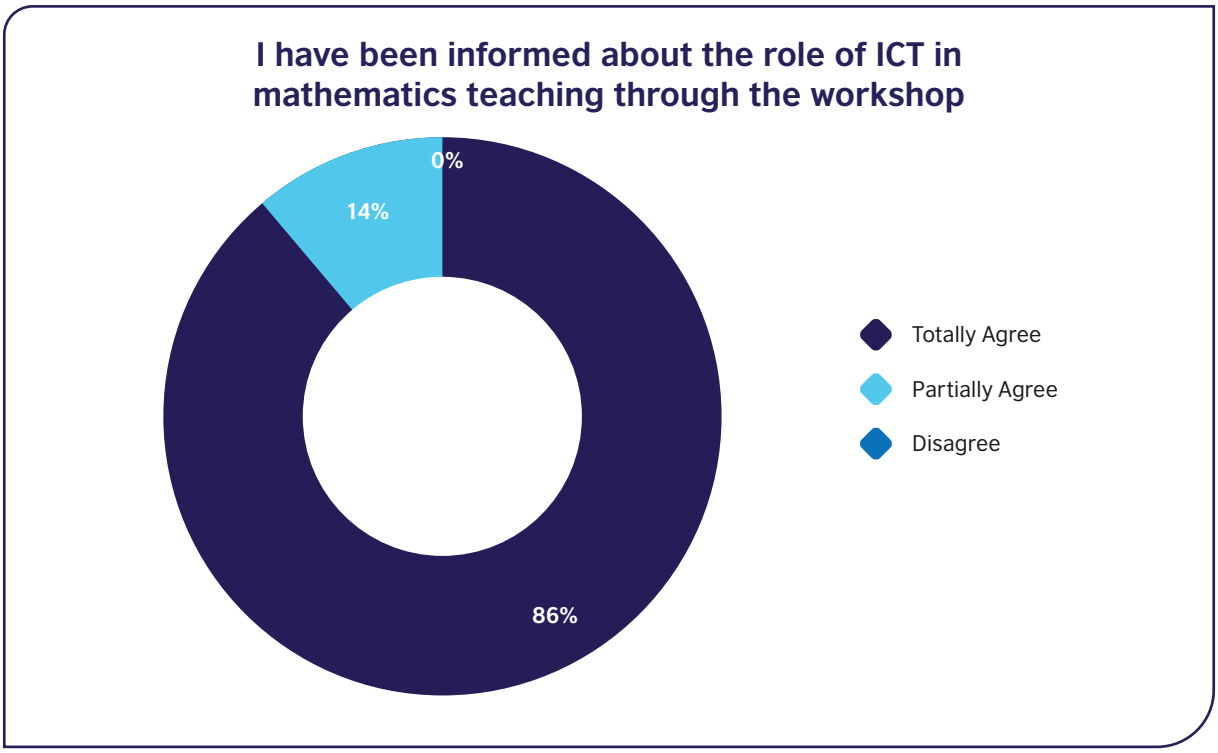


Figure 19: Response to the Question 5.

Question 4 was totally agreed upon by 100% of the participants. 86% of the participants agreed totally with Question 5 whereas 14% agreed partially with the question

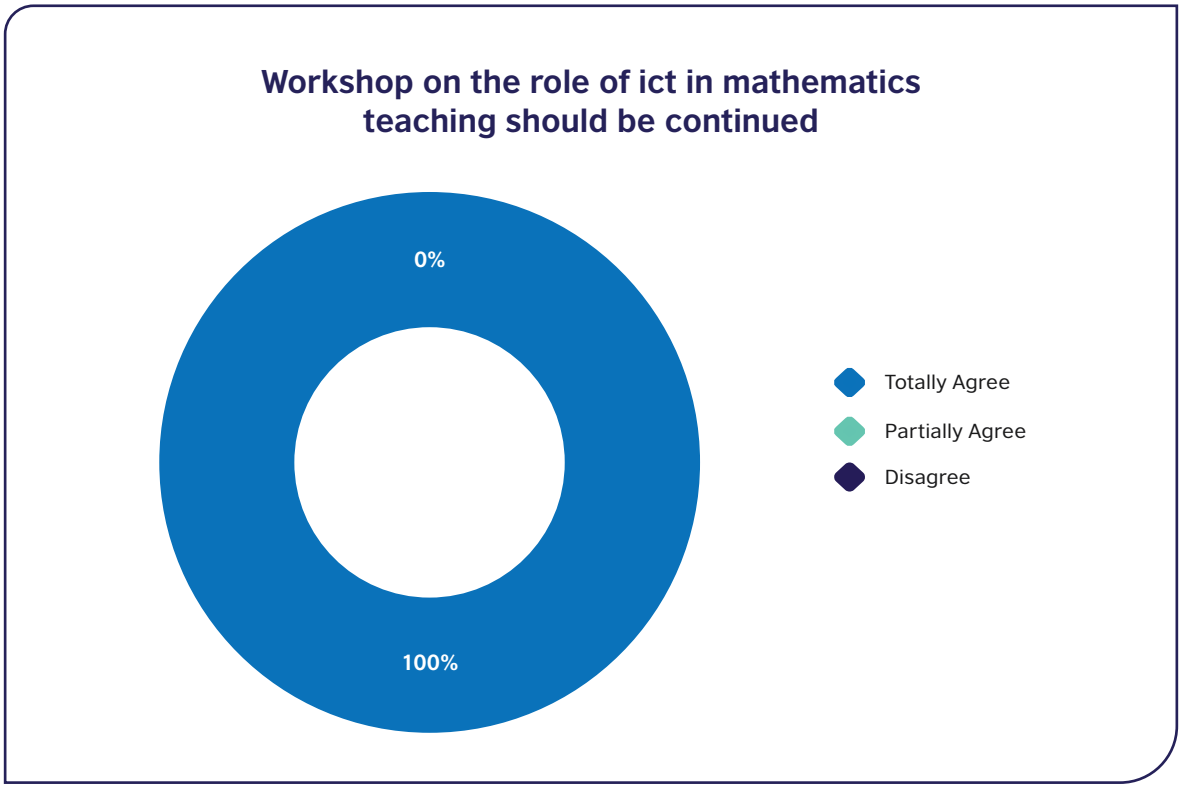


Figure 20: Response to the Question 6.

All the participants agreed totally with Question 6: such type of workshop should be organized regularly

6.2 Training Workshop at the Kishoreganj District

A training workshop on “Teaching and Learning Mathematics using Computer Algebra System (CAS) at Secondary Level” was organized on 22 May 2022 at the Abdul Aziz High School, Kishoreganj. 28 mathematics teachers from di erent secondary institutions participated in the workshop and given their feedbacks.

6.2.1 Participants of the Workshop

The participants of the workshop are listed in the following table:

Name	Qualification	School
Md. Ashraful Islam	B.Sc., B.Ed.	Garh Bishudia B Noich de Madrasa
Md. Abul Khair	B.Sc., B.Ed.	Mechera High School
Rafiqul Islam	M.Sc.	Pumdi Islamia High School
Md. Abdul Quader Sheikh	B.Sc., B.Ed.	Pitalganj High School
Md. Ruhul Amin	B.Sc., B.Ed.	Akbar Ali Technical School & College
Md. Hafeez Uddin	B.Sc., B.Ed.	Hossainpur Upazila Badar Madrasa
Md. Dulal Uddin	B.Sc., B.Ed.	Rampur Hoduinia Balika Madrasa
Saiful Islam	M.Sc.	SRD Shamsuddin Bhuiyan School & College
Md. Asaduzzaman	B.Sc. , B.Ed.	Abdul Aziz High School
Md. Abu Shahadat Chayem	B.Sc., B.Ed.	Charkathihari M Ali Lower Secondary School
Nasreen Akhter Khatun	B.Sc., B.Ed.	Charhajira High School
Taslima Akhter	B.Sc., M.Sc.	Piplakandi Girls High School
Md. Mujibur Rahman	M.Sc., BSc, B.Ed.	Madmala Zayil Madrasa
Md. Rafiqul Islam	B.Sc., B.Ed.	Hassanpur Govt. Model Pilot School
Md. A. Malek Akand	B.Sc., B.Ed.	Dr. Samsuddin Khan Girls High School
Abul Kalam Ilias	B.Sc., M.Sc.	Hossainpur Pilot Girls High School
Md. Humayun Kabir	B.Sc., B.Ed.	Shamsul Islam Ideal Girls High School
Md. Shafiqul Islam	B.Sc.	Ahmad Zubaidah Islamia Madrasa
Md. Sohag Mia	B.Sc., B.Ed.	Ashutia Damadia Dakhil Madrasa
Azizul Haque	M.Sc.	Abdul Aziz High School
Md. Tafazzul Hossain	B.Sc., B.Ed.	Galachipa Secondary School
Monirzaman	M.Sc., B.Ed.	Haji Jalal A. Ubi: Hossainpur
Meftahul Jannat	B.Sc., B.Ed.	Shahedal Daruchunnah Madrasa
Md. Zahirul Alam	M.Sc.	Syedunnesa Alim Madrasa
Md. Mahmudul Hasan	M.Sc.	Pobindapur High School
Md. Ashraful Islam	B.Sc., M.Sc.	Syedpur Hasania Dakhil Madrasa
Md. Al-Amin	M.Sc.	Lulikandi High School
Md. Abu Saeed	B.Sc, B.Ed.	Dapunia Ahmadu JAD Madrasa

Academic qualifications of the participants in the workshop can be figured in the following manner: 57% of the participants have the qualification B.Sc., 7% of the participants have B.Sc. with B.Ed., 29% have M.Sc. degree, and 7% have M.Sc. degree with B.Ed. degree as presented in the following diagram.

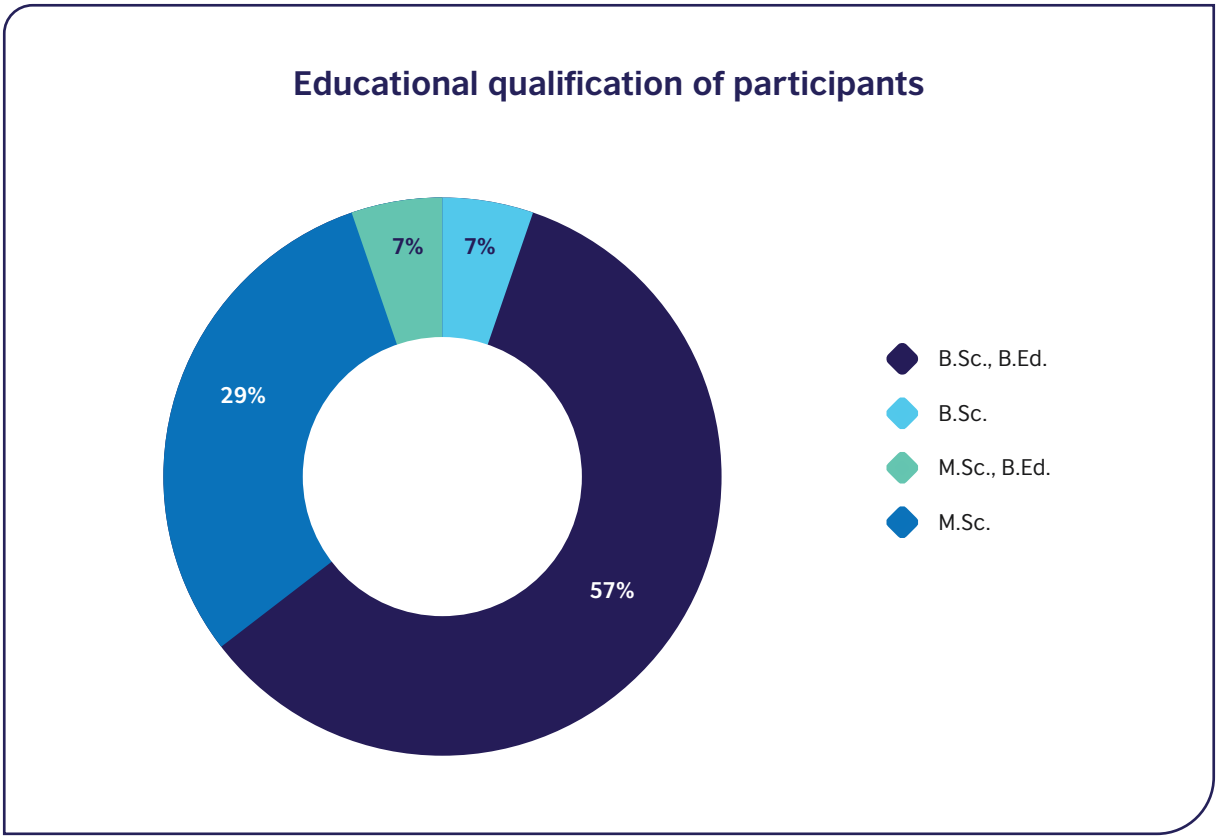


Figure 21: Academic Degree of the Participants in the Workshop.

6.2.2 Feedback from the Participants

At the end of the workshop, a feedback form was filled out by all the participants in that workshop that consists of the following questionnaire:
Q1. Recently, reluctance towards mathematics education is being observed among stu dents at the secondary level.

Q2. The use of ICT is important to ensure quality mathematics teaching at the secondary level.

Q3. Computer Algebra System (CAS) will play a significant role in alleviating students' math fears and anxiety.

Q4. The use of ICT can facilitate graph sketching at the secondary level of mathematics.

Q5. I have been informed about the role of CAS-aided mathematics teaching through this workshop.

Q6. Training workshops on the role of CAS-aided mathematics teaching should be continued.

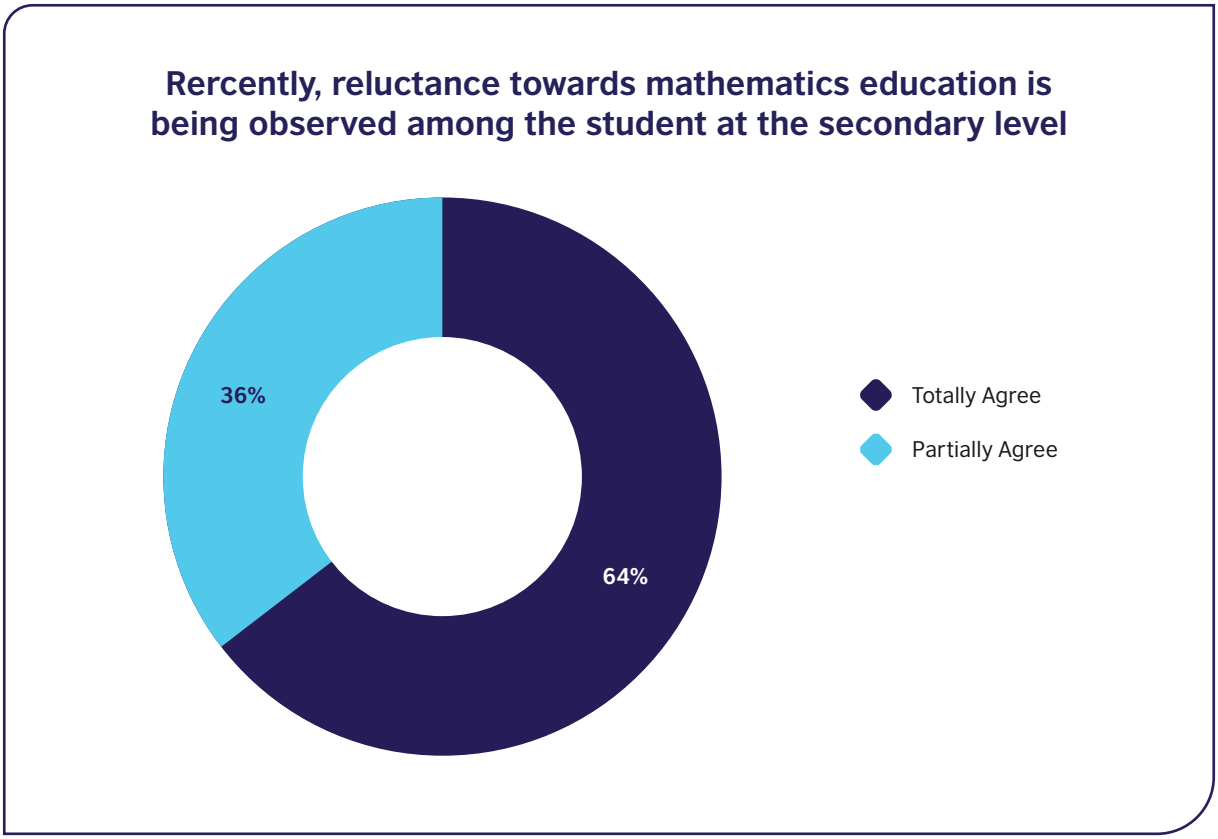


Figure 22: Response to the Question 1.

It was observed that 64% of the participants totally agreed with (Q1) and 36% agreed partially with the question.

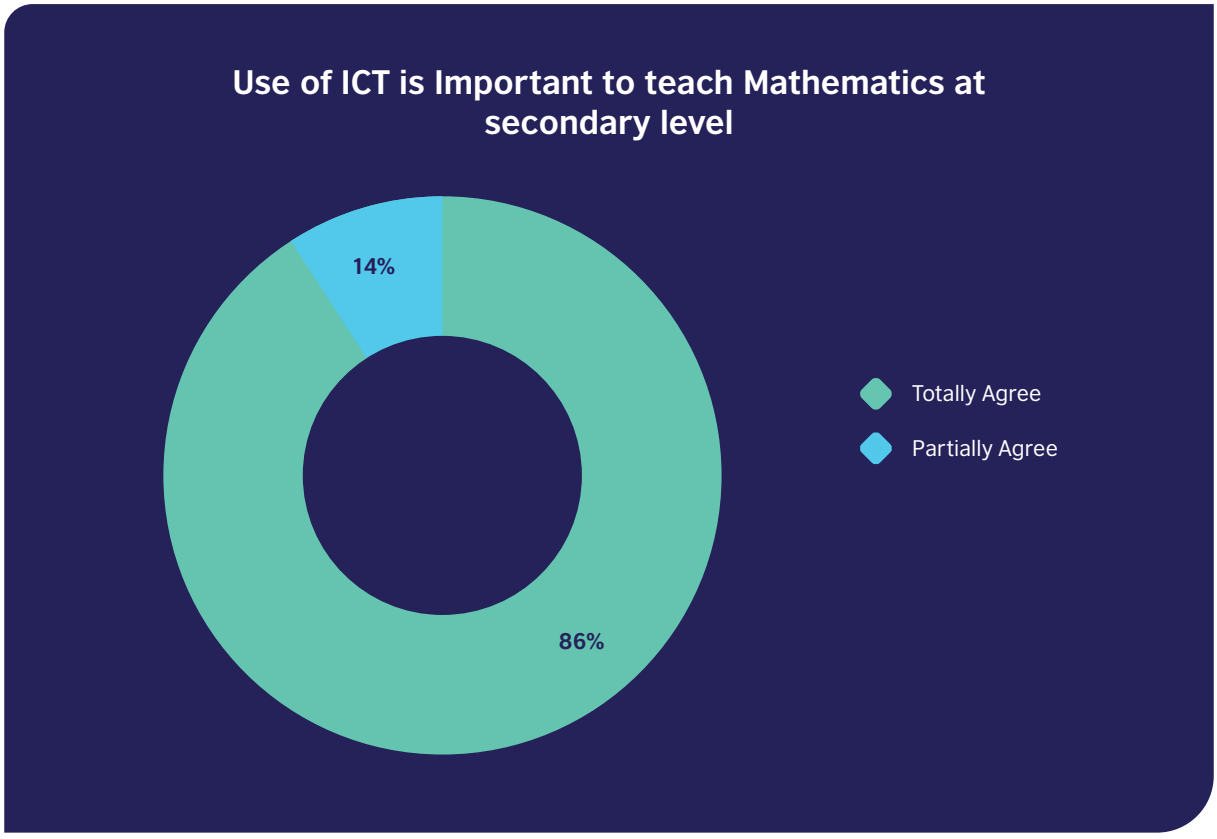


Figure 23: Response to the Question 2.

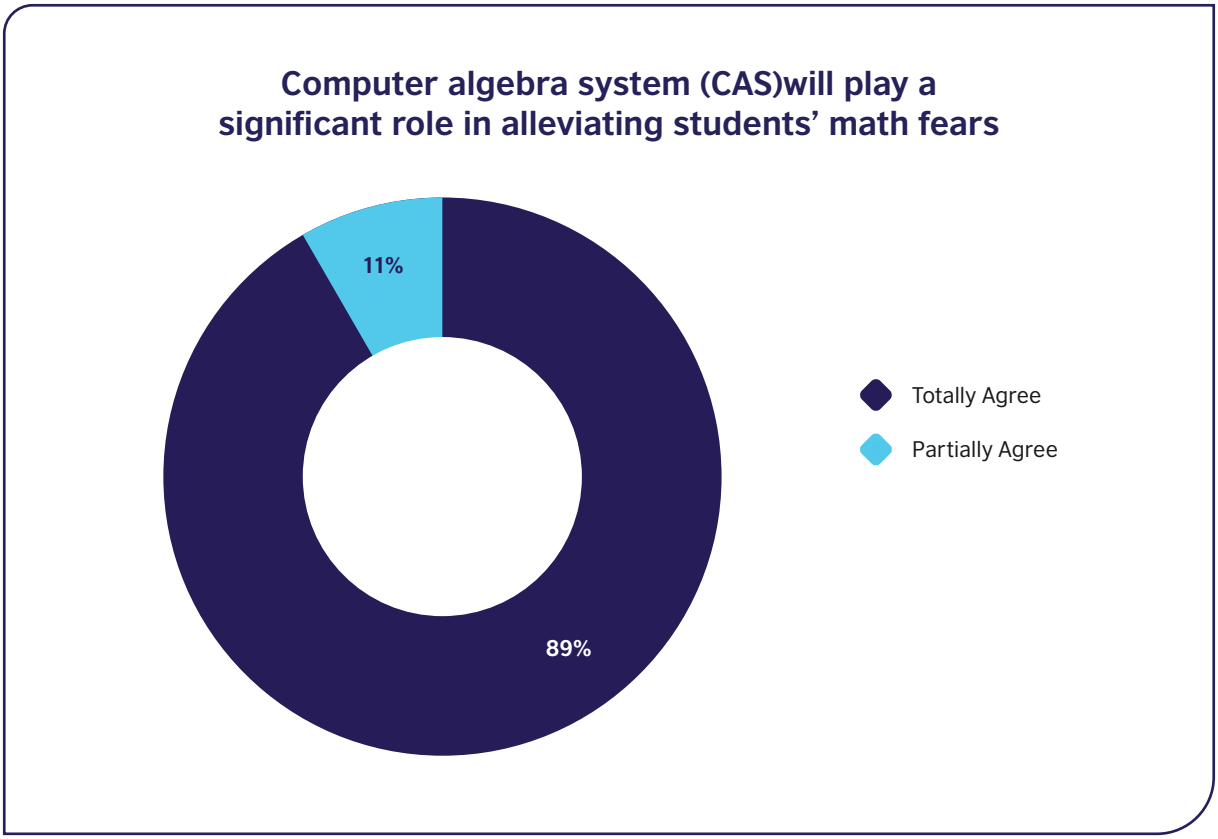


Figure 24: Response to the Question 3.

Question 2 was totally agreed by 86% of the participants and 14% of the participants partially agreed. 89% of participants agreed totally with question 3 while 11% agreed partially.

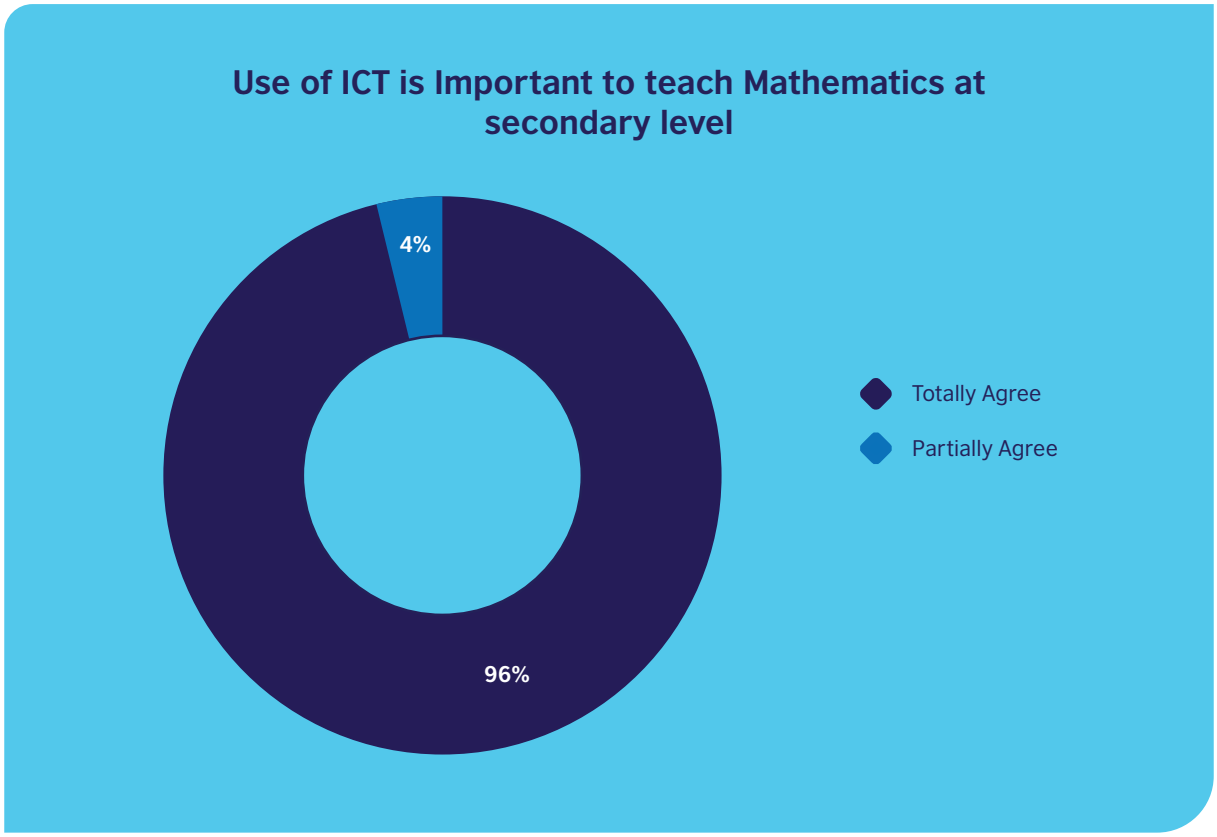


Figure 25: Response to the Question 4.

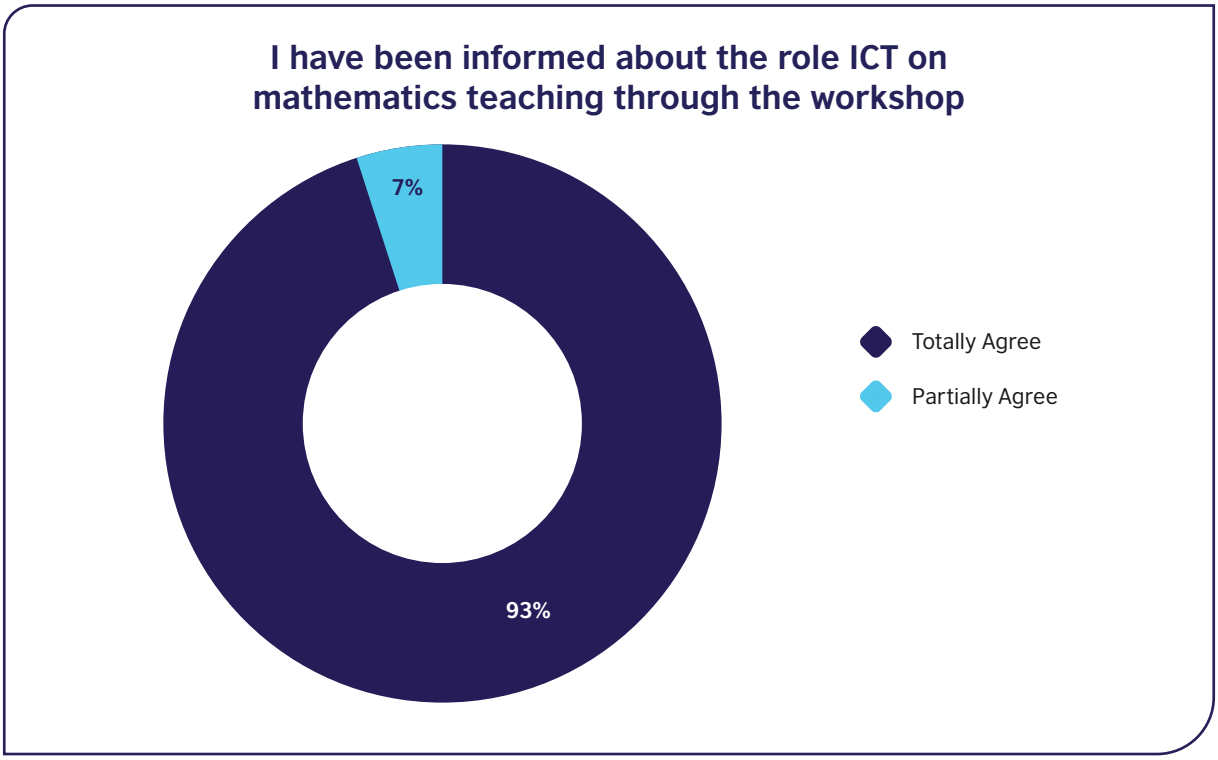


Figure 26: Response to theQuestion 5.

Question 4 was totally agreed by 93% of the participants and 4% partially agreed. 93% of the participants agreed totally with Question 5 whereas 7% agreed partially with the question.

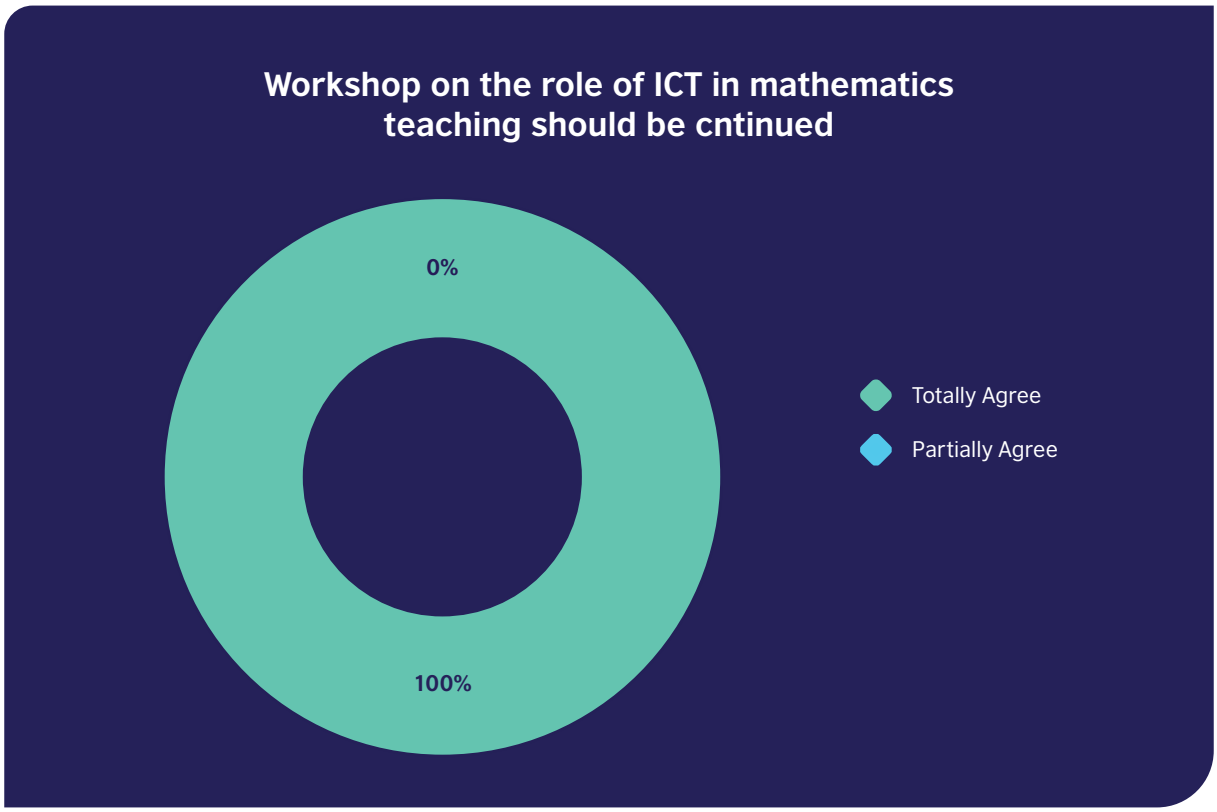


Figure 27: Response to the Question 6.

All the participants agreed totally with Question 6: such type of workshop should be continued.

6.3 Training Workshop at the Dhaka District

A training workshop on "Teaching and Learning Mathematics using Computer Algebra System (CAS) at Secondary Level" was organized on 26 June 2022 at the department of Mathematics, Jahangirnagar University, Dhaka. 41 mathematics teachers from different secondary institutions participated in the workshop and put their feedbacks.

6.3.1 Participants of the Workshop

The participants of the workshop are listed in the following table:

Name	Qualification	School
Md. Showkat Hossain	B.Sc., M.Sc.	BPATC School & College
Md. Golam Moula	M.Sc., B.Ed.	Kolotan Biddaniketon
Nishat Sultana Bithy	M.Sc.	BPATC School & College
Asma Ahmed	B.Sc., M.Sc.	BPATC School & College
Md. Abu Bakkar Siddik	B.Sc., M.Sc., B.Ed.	Radio Colony Model School & College
Md. Abdush Salam Mia	B.Sc., B.Ed.	Radio Colony Model School & College
Md. Jashim Uddin	B.Sc., B.Ed.	Radio Colony Model School & College
Md. Manek Mia	B.Sc., M.Sc.	Radio Colony Model School & College
Md. Mahfuz Alam	B.Sc., M.Sc.	Radio Colony Model School & College
Md. Majbaul Haque Mithun	B.Sc.	Milestone School & college
Md. Taherul Zaman	B.Sc., B.Ed.	Jahangirnagar University School & College
Rumana Parven	M.Sc.	Jahangirnagar University School & College
Rimon Saruare	M.Sc.	Shena Public School & College
Md. Hafezul Islam	B.Sc., M.Sc.	Shena Public School & College
Robiul Aual	B.Sc.	Shena Public School & College
ABM Golam Mostafa	B.Sc., M.Sc.	Savar Coatt Public School & College
Md. Shamsur Rahman	B.Sc., B.Ed.	Shena Public School & College
Md. Assaduzzaman	M.Sc.	Shena Public School & College
Tufazzal Hossain	B.Sc., M.Sc	Shena Public School & College
Md. Atuar Rahman Ovi	M.Sc.	Savar Cantonment Public School & College
Mehedin Hasan Mamun	B.Sc.	Savar Cantonment Public School & College
Md. Saiful Islam	M.Sc.	Shena Public School & College
Md. Rashed Al Mamun	M.Sc.	Savar Cantonment Public School & College
Md. Shahajul Islam	B.Sc.	Savar Cantonment Public School & College
Md Fahim Hasan	B.Sc., M.Sc.	Savar Cantonment Public School & College
Remaul Islam	M.Sc.	Savar Cantonment Public School & College
Md. Mahbub Alam	M.Sc., B.Ed.	Savar Cantonment Public School & College
Md. Robiul Hassan	M.Sc.	Savar Cantonment Public School & College
Md. Rofiqul Islam	M.Sc.	Savar Cantonment Public School & College
Farjana Karim	M.Sc.	Savar Govt Odhor Chandra High School
Kamrun Nahar	B.Sc., B.Ed.	Savar Govt Odhor Chandra High School
Md. Abul Quayum	B.Sc., M.Sc	Savar Govt Odhor Chandra High School
Md. Zohurul Islam	M.Sc.	Savar Govt Odhor Chandra High School
Nandini Dey	B.Sc., M.Sc.	Savar Laboratory School
Md. Moniruzzaman	M.Sc.	Savar Laboratory School
Md. Anwar Hossain	B.Sc., M.Sc.	Savar Laboratory School
Md. Nazmul Huda	B.Sc., M.Sc.	Savar Laboratory School
Md. Golam Shakhawate	M.Sc.	Green Bell Laboratory School
A.T.M Halimuzzaman	M.Sc.	Green Bell Laboratory School
Subrata Chandra Das	M.Sc.	Collegiate School
Nazma Akther Lucky	M.Sc.	Collegiate School

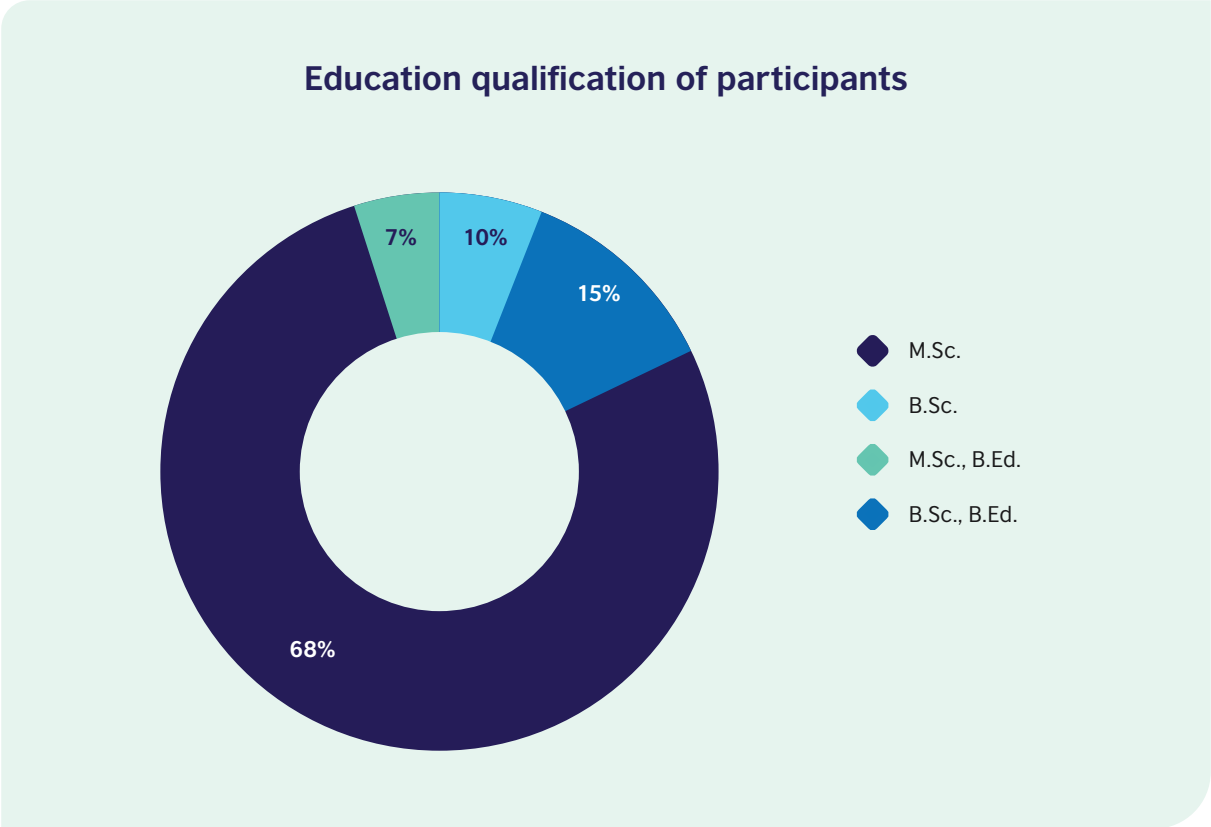


Figure 28: Academic Degree of the Participants in the Workshop.

Academic qualifications of the participants in the workshop can be figured in the following manner: 10% of the participants have qualification B.Sc., 15% of the participants of B.Sc. with B.Ed., 68% have M.Sc., and 7% have M.Sc. degree with B.Ed. as presented in the above diagram.

6.3.2 Feedback from Participants

At the end of the workshop, a feedback form was filled out by all the participants in that workshop that consists of the following questionnaire:

Q1. Recently, reluctance towards mathematics education is being observed among the students at the secondary level.

Q2. The use of ICT is important to ensure quality mathematics teaching at the secondary level.

Q3. Computer Algebra System (CAS) will play a significant role in alleviating students' math fears and anxiety.

Q4. The use of ICT can facilitate graph sketching at the secondary level of mathematics.

Q5. I have been informed about the role of the CAS-aided mathematics teaching through this workshop.

Q6. Training workshops on the role of the CAS-aided mathematics teaching should be continued.

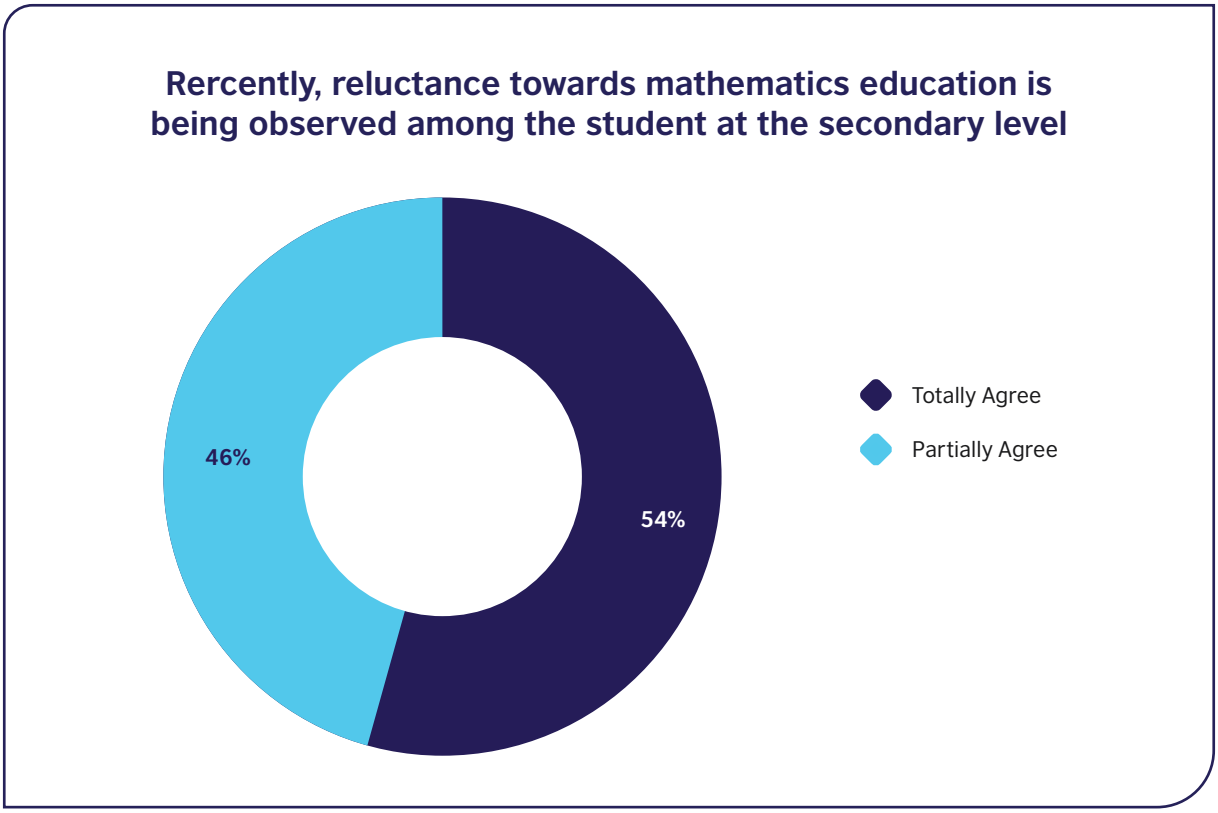


Figure 29: Response to the Question 1.

It was observed that 54% of the participants totally agreed with (Q1) and 46% agreed partially with the question.

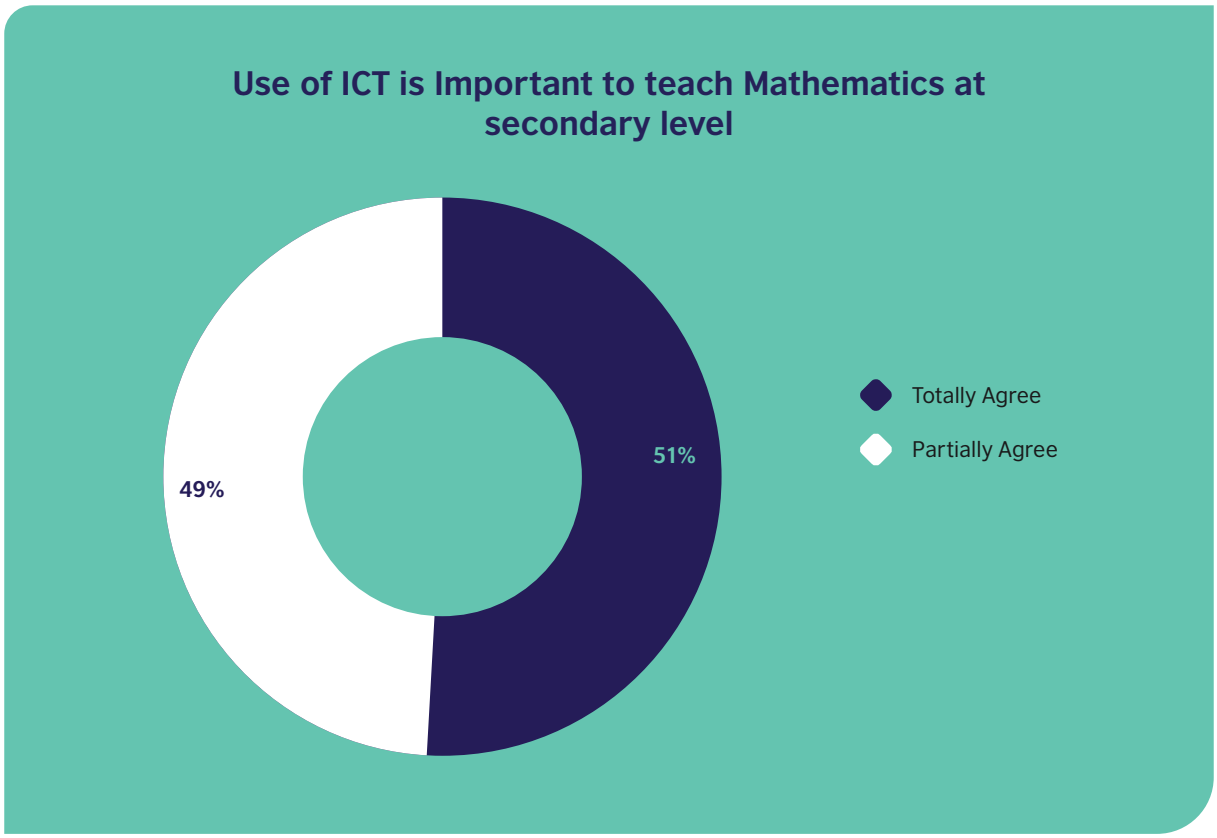


Figure 30: Response to the Question 2.

Question 2 was totally agreed by 51% of the participants and 49% of the participants partially agreed. 89% of participants agreed totally with question 3 while 11% agreed partially.

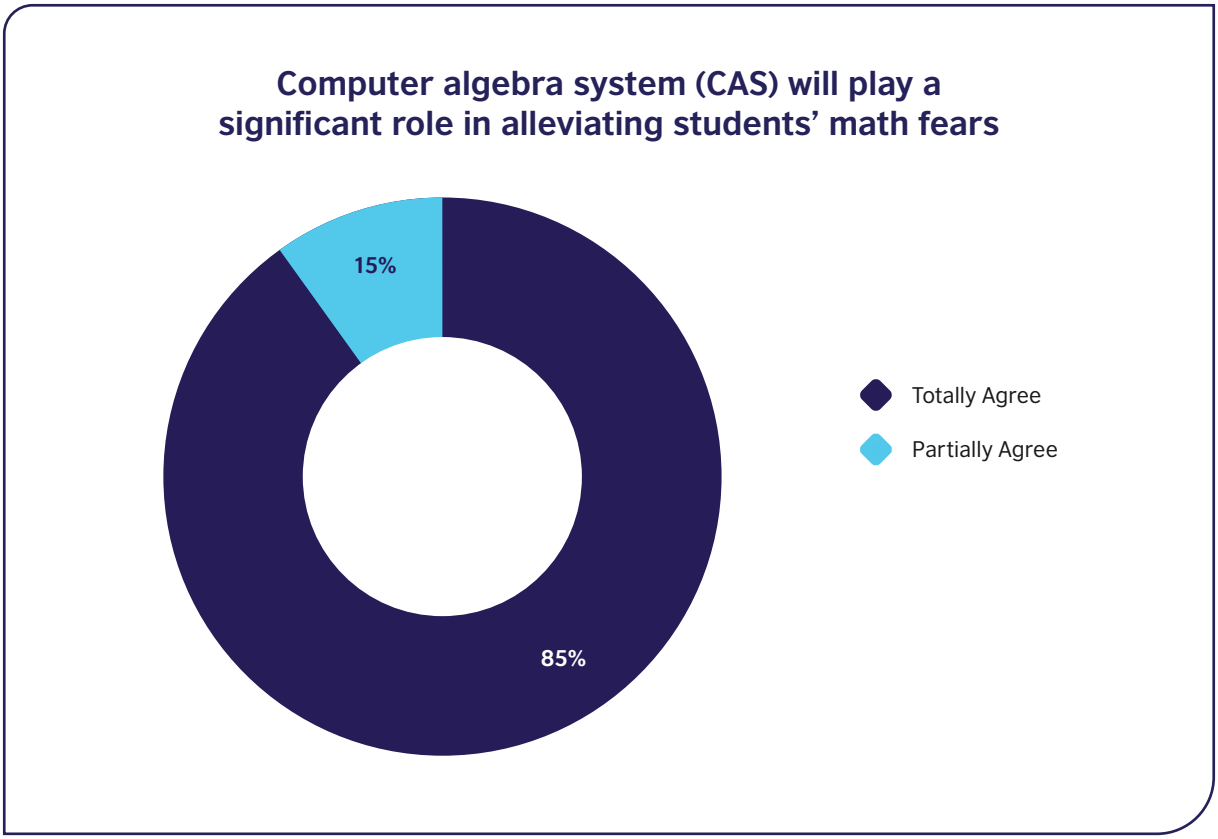


Figure 31: Response to the Question 3.

Question 3 was totally agreed by 85% of the participants and 15% of the participants partially agreed.

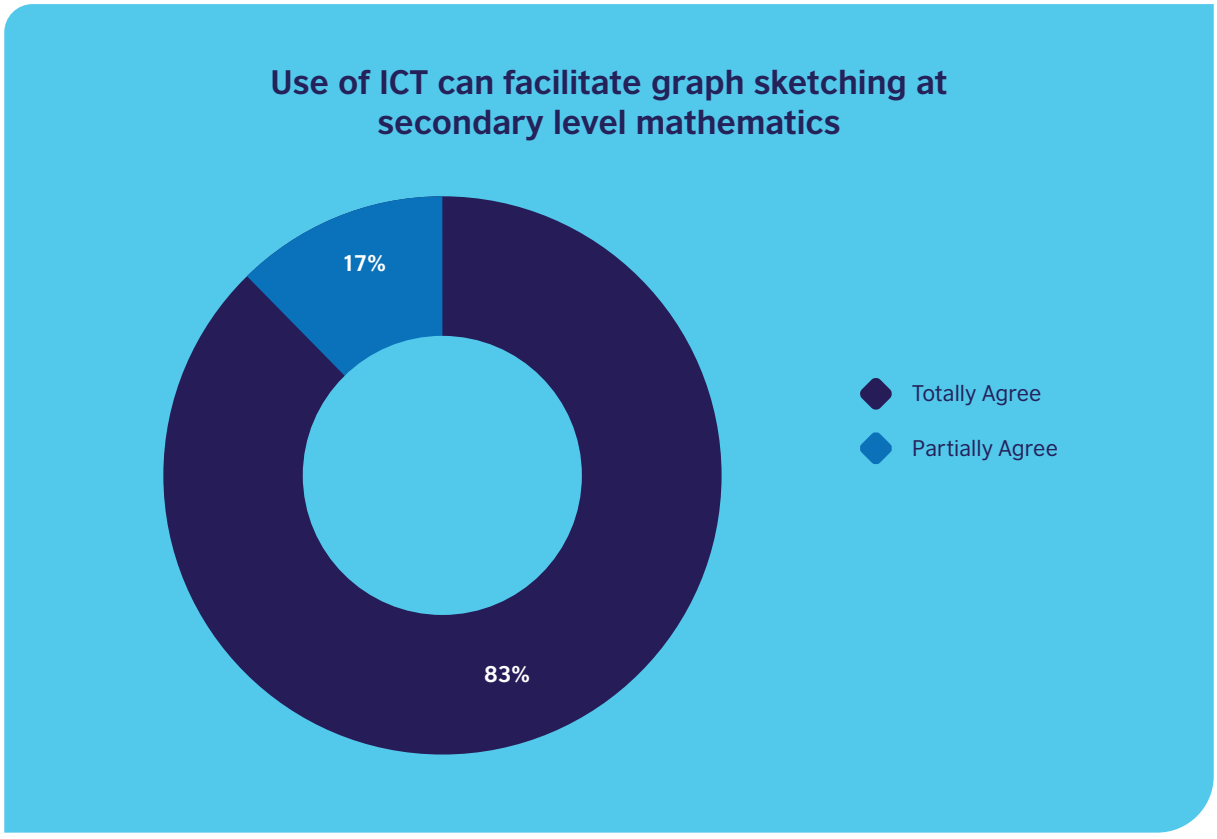


Figure 32: Response to the Question 4.

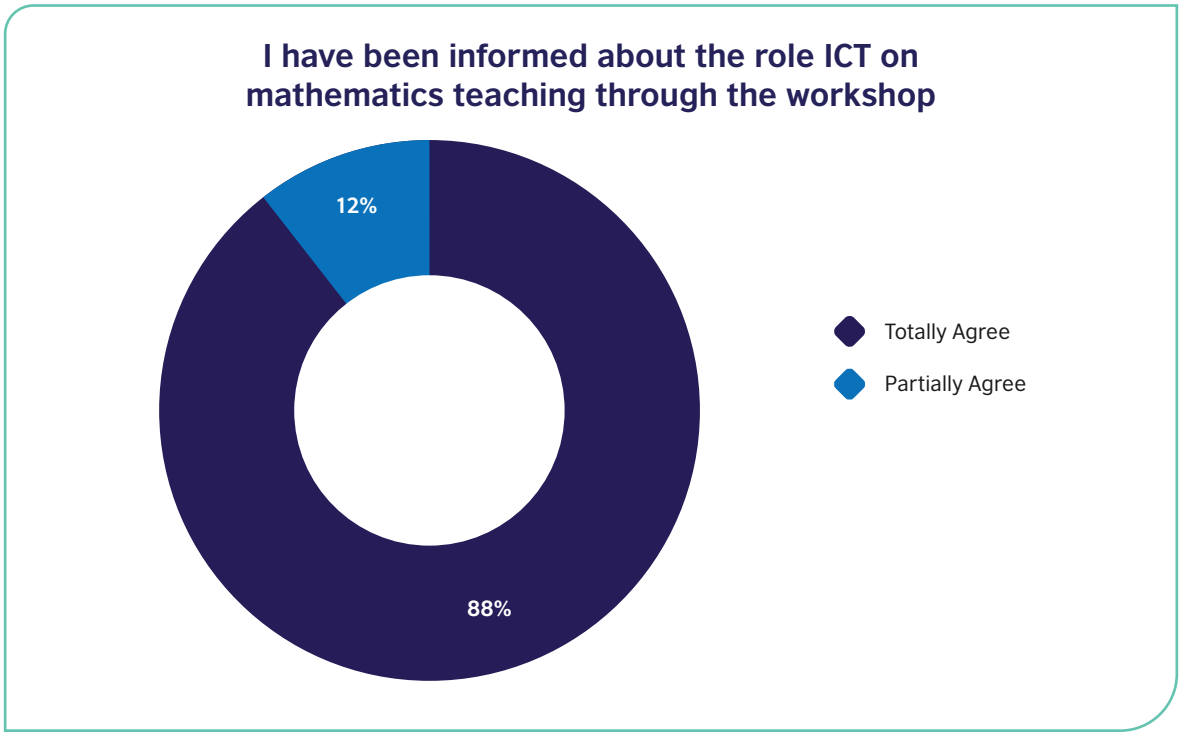


Figure 33: Response to the Question 5.

Question 4 was totally agreed by 83% of the participants and 17% partially agreed. 88% of the participants agreed totally with Question 5 whereas 12% agreed partially with the question. All the participants agreed totally with Question 6: such type of workshop should be continued.

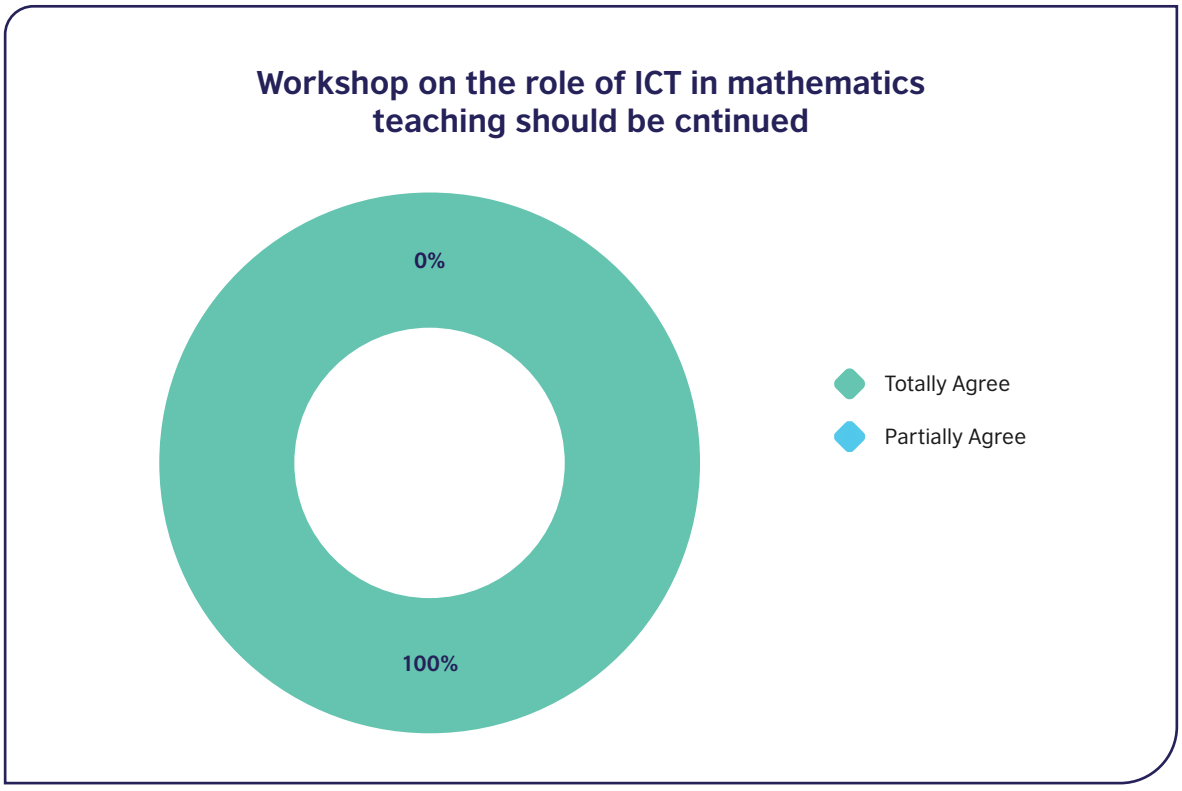


Figure 34: Response to the Question 6.

To categorize gender participation in the three training workshops, we present the statis tics of the participants concerning gender in the following table. It is observed that more than 87% participants in the workshop are male while 13.1% are female, as presented in Figure 35. Women's participation in mathematics teaching is not so satisfactory according to the study.

Place of the Workshop	Number of Male	Number of Female	Total Participants
Narsingdi	34	4	38
Kishorganj	25	3	28
Savar	34	7	41
Total	**93**	**14**	**107**

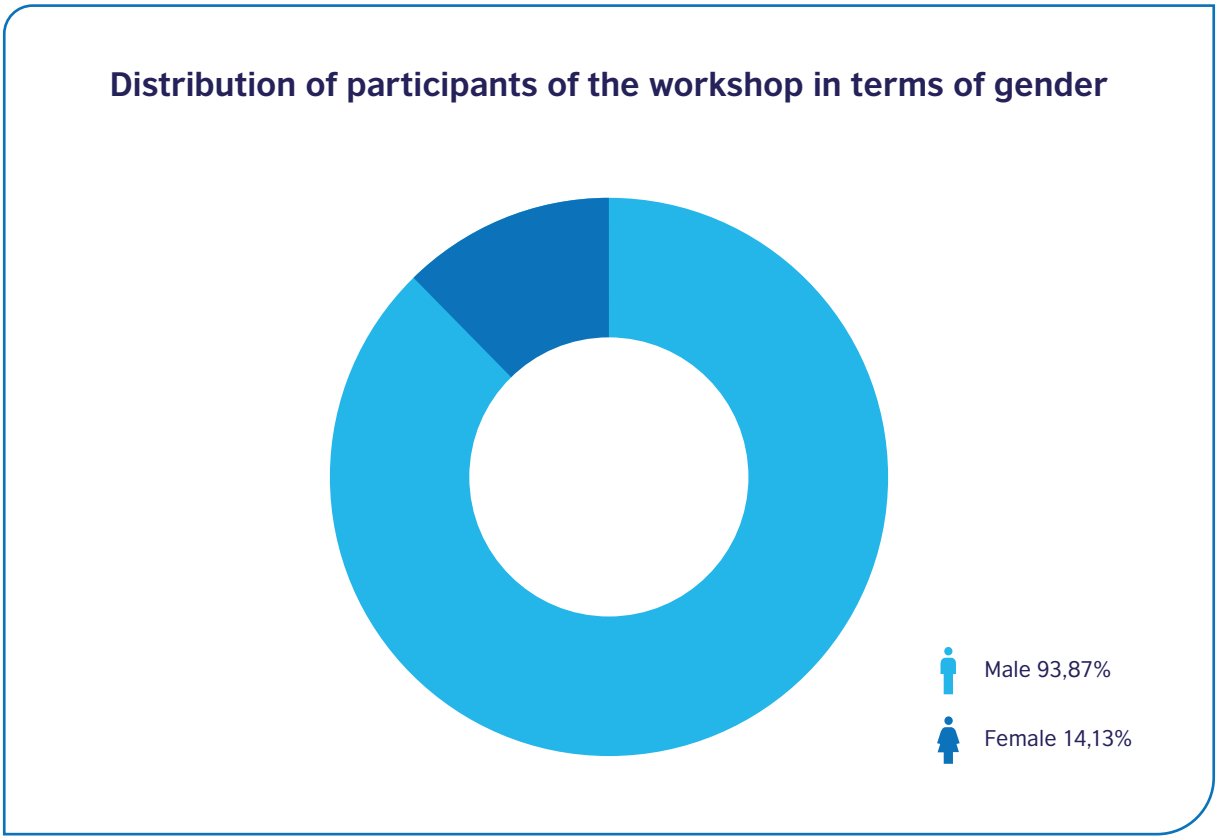


Figure 35: Distribution of participation in terms of gender.

6.4 Snapshots during the Workshops

A nice gathering and interactive participation are observed in the workshop that was held in three different areas. Some selected snapshots of the workshops are presented here.

Figure 36: Snapshot of the workshop.

Figure 37: Snapshot of the workshop.

6.5 Feedback Comments provided by the Participants

It is perceived that all the participants of the workshops passed a few common comments on the CAS-aided mathematics teaching and learning which are very important to judge the impression of the teachers regarding the implementation of such a scientific tool into the secondary mathematics curriculum. Some common comments are listed in the following table.

Comments made by the participants of the workshops
1. CAS-aided mathematics teaching is extremely important and essential for enhancing students' mathematics proficiency.
2. Such a technological adaptation can make mathematics learning enjoyable.
3. ICT support in secondary educational institutions should be increased because CAS-aided mathematics teaching can be performed smoothly.
4. Mathematics teachers should be trained regularly by an expert in CAS aided mathematics teaching and learning.
5. Training workshops on CAS-aided mathematics teaching should be organized regularly not only in urban and semi-urban areas but also in rural areas

CHAPTER 7
Discussions and Recommendations

7.1 Concluding Remarks

The research on ICT-aided mathematics teaching aims to understand the existing scenario and how teachers use ICT to teach mathematics at the secondary level. The research project has consisted of two parts: (i) a survey on ICT-aided mathematics teaching and (ii) a training workshop to introduce CAS (e.g. GeoGebra) to mathematics teachers at the secondary level. The study along with other peripheral issues examined 2 (two) spe cific indicators such as teachers' qualification and training (Pre-service and in-service), covering more than 200 secondary educational institutions located in the rural and urban areas of six divisions. It was a sample study identifying qualifications (those who are employed as Mathematics teachers) as a central indicator for achieving the goal of quality secondary mathematics education in Bangladesh. The survey has revealed some interesting and implementable results. It reflects that there is an affordable opportunity for incorporating CAS by providing adequate teacher training on "CAS-aided mathematics teaching". The outcomes of this research are indicative of the complete scenario across the country and can help in developing a sustainable education policy. The survey identified that in some cases mathematics teachers lacked requisite qualifications and sufficient training and there is a need to enhance the teaching and training strength of these teachers. Due to rapid change in curriculum, further study at a bigger scale will give a clear and more dependable impression of the complete scenario.

The major findings of the research project are as follows:

• 100% mathematics teachers thinks that implementation of ICT is mighty important to teach at the secondary level effectively.

• 97.7% teachers do not use any CAS package to teach mathematics at the secondary level while 2.3 % are using CAS package in a limited platform.

• Only 9% mathematics teachers have ICT training and 91% have no ICT training at all. The in-service ICT training for mathematics teachers is quite adequate to make them competent in the subject.

• 100% teachers are willing to attend a training workshop on CAS-aided mathematics teaching to learn such a technological teaching tool.

• An ICT laboratory is available in almost 98% secondary institutions that can be used to conduct CAS-aided mathematics teaching.

• 23% participants of the workshop have a B.Sc. with B.Ed. degree, 17.7% have an M.Sc. with B.Ed. degree, 32.3% have the M.Sc. degree, and 22% have the B.Sc. degree only. This has resulted in less than-expected proficiency of the teachers.

• 60% participants of the workshop agree totally that the students have math anxiety nowadays while 38% agree partially with the statement.

• 94% of teachers feel that CAS could play a significant role in alleviating students' math anxiety.

• 100% participants perceived that CAS, e.g. GeoGebra could facilitate the understanding of hard and sophisticated math problems.

• 100% teachers commented that such a training workshop on CAS-aided mathematics teaching should be continued.

• Only 13% female mathematics teachers are serving at the secondary level.

7.2 Recommendations

To perform CAS-aided Mathematics teaching, we propose the following recommendations based on the findings of the research

• Mathematics Teachers should be trained adequately for CAS-aided mathematics teaching

• ICT support in secondary school should be enhanced so that CAS can be incorporated mathematics curriculum smoothly

• A CAS package, e.g. GeoGebra should be imposed in the Mathematics curriculum at the secondary level

• Gender equality should be ensured by increasing female participation in mathematics teaching.

• Internet packages should be subsidized for teaching and learning at the lowest cost.

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Appendix A

BRITISH COUNCIL

Training Workshop
on
Teaching and Learning Mathematics using Computer Algebra System (CAS) at Secondary Level

জরীপ প্রশ্নাবলী

১. নাম :
২. শিক্ষাগত যোগ্যতা :
৩. প্রতিষ্ঠানের নাম :
৪. মোবাইল :
৫. ইমেইল :

(নির্দিষ্ট বক্সে টিক চিহ্ন দিন)

১. আপনি কি মনে করেন মাধ্যমিক পর্যায়ে গণিত শেখানোর জন্যে আইসিটি এর ব্যবহার গুরুত্বপূর্ণ?
☐ হ্যাঁ ☐ না

২. আপনি কি গণিত শেখানোর জন্যে কম্পিউটার অ্যালজেবরা সিস্টেম (CAS) এর কোনো প্যাকেজ ব্যবহার করেন?
☐ হ্যাঁ ☐ না

৩. মাধ্যমিক পর্যায়ে গণিত শেখানোর জন্যে আপনি কি আইসিটি-সম্পর্কিত কোনো সমস্যার সম্মুখীন হচ্ছেন?
☐ হ্যাঁ ☐ না

৪. আপনি কি ইতিপূর্বে আইসিটি-সহায়ক গণিত শিক্ষার বিষয়ক কোনো প্রশিক্ষণ গ্রহণ করেছেন?
☐ হ্যাঁ ☐ না

৫. আইসিটির সহায়তায় গণিত শিক্ষা বিষয়ক প্রশিক্ষণ কর্মশালায় অংশগ্রহণ করতে আগ্রহী?
☐ হ্যাঁ ☐ না

৬. মন্তব্য

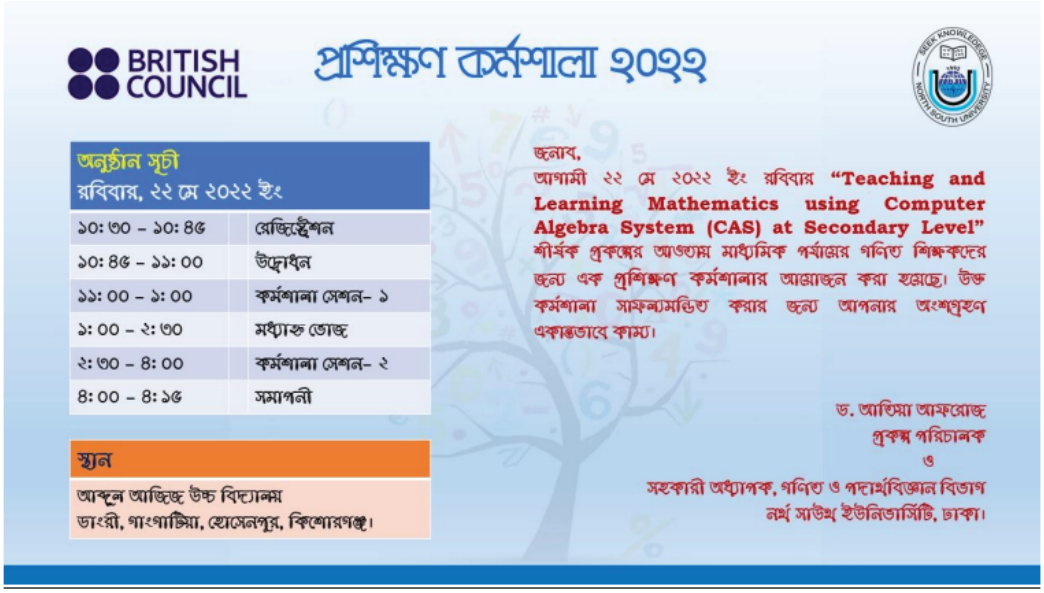
স্বাক্ষর

Appendix B

BRITISH COUNCIL

Training Workshop
on
Teaching and Learning Mathematics using Computer Algebra System (CAS) at Secondary Level

কর্মশালা মডিউল



BRITISH COUNCIL

প্রশিক্ষণ কর্মশালা ২০২২

অনুষ্ঠান সূচী রবিবার, ২২ মে ২০২২ ইং	
১০: ৩০ – ১০: ৪৫	রেজিস্ট্রেশন
১০: ৪৫ – ১১: ০০	উদ্বোধন
১১: ০০ – ১: ০০	কর্মশালা সেশন- ১
১: ০০ – ২: ৩০	মধ্যাহ্ন ভোজ
২: ৩০ – ৪: ০০	কর্মশালা সেশন- ২
৪: ০০ – ৪: ১৫	সমাপনী

স্থান
আব্দুল আজিজ উচ্চ বিদ্যালয়
ডাংরী, গাংগাচিয়া, হোসেনপুর, কিশোরগঞ্জ।

জনাব,
আগামী ২২ মে ২০২২ ইং রবিবার **"Teaching and Learning Mathematics using Computer Algebra System (CAS) at Secondary Level"** শীর্ষক প্রকল্পের আওতায় মাধ্যমিক পর্যায়ের গণিত শিক্ষকদের জন্যে এক প্রশিক্ষণ কর্মশালার আয়োজন করা হয়েছে। উক্ত কর্মশালা সাফল্যমন্ডিত করার জন্যে আপনার অংশগ্রহণ একান্তভাবে কাম্য।

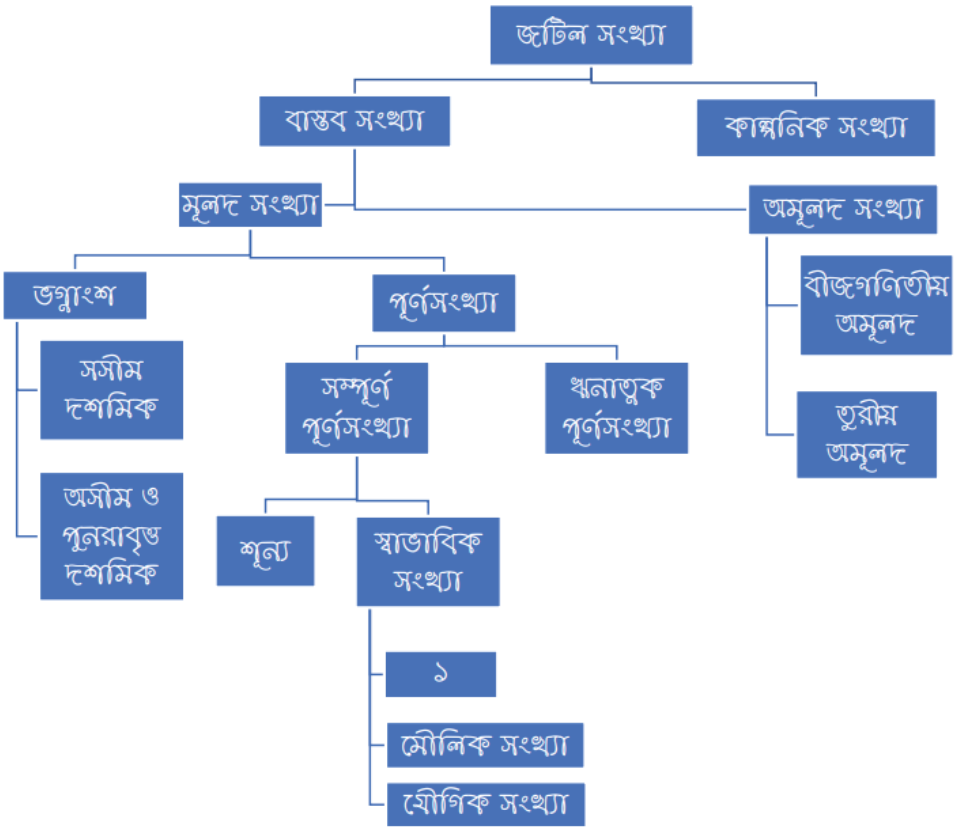
ড. আতিয়া আফরোজ
প্রকল্প পরিচালক
ও
সহকারী অধ্যাপক, গণিত ও পদার্থবিজ্ঞান বিভাগ
নর্থ সাউথ ইউনিভার্সিটি, ঢাকা।

প্রশিক্ষণ বিশেষজ্ঞ

ড. মোহাম্মদ হুমায়ুন কবীর
সহযোগী অধ্যাপক, গণিত বিভাগ
জাহাঙ্গীরনগর বিশ্ববিদ্যালয়, ঢাকা।

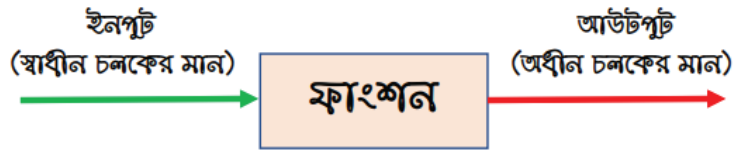
১. সংখ্যা (Number)

সংখ্যা হল একটি গাণিতিক মান যা পরিমাণকে প্রকাশ করে এবং গণনা করার জন্যে ব্যবহৃত হয়। খ্রিস্টপূর্ব ৩৪০০ সালে টাইগ্রিস ও ইউফ্রেটিস নদীর তীরে (বর্তমান ইরাক ও কুয়েত) মেসোপটেমিয়ান সভ্যতার সময়কালে সর্বপ্রথম ষষ্ঠিক (Sexagesimal) সংখ্যার প্রচলন হয়। খ্রিস্টপূর্ব ৩১০০ সালে মিশরে (Decimal) দশমিক সংখ্যার প্রবর্তন হয়। কালক্রমে ৮ম ও ৯ম শতাব্দীতে স্বাভাবিক সংখ্যার (Hindu-Arabic Numerals) গোড়াপত্তন হয়। পরবর্তীতে বৈশিষ্ট্যের ভিন্নতার ভিত্তিতে সংখ্যাকে বিভিন্ন প্রকারে শনাক্ত করা হয়, যা নিম্নোক্ত কাঠামো দ্বারা প্রকাশ করা হয়।



২. ফাংশন, ডোমেন এবং রেঞ্জ

ফাংশন (Function): যদি দুইটি চলক এমনভাবে সম্পর্কযুক্ত থাকে যেন স্বাধীন চলকের একটি মানের জন্যে অধীন চলকের একটি কেবলমাত্র একটি মান পাওয়া যায়, তখন উক্ত সম্পর্ককে ফাংশন বলা হয়, যেমন $y = f(x)$.



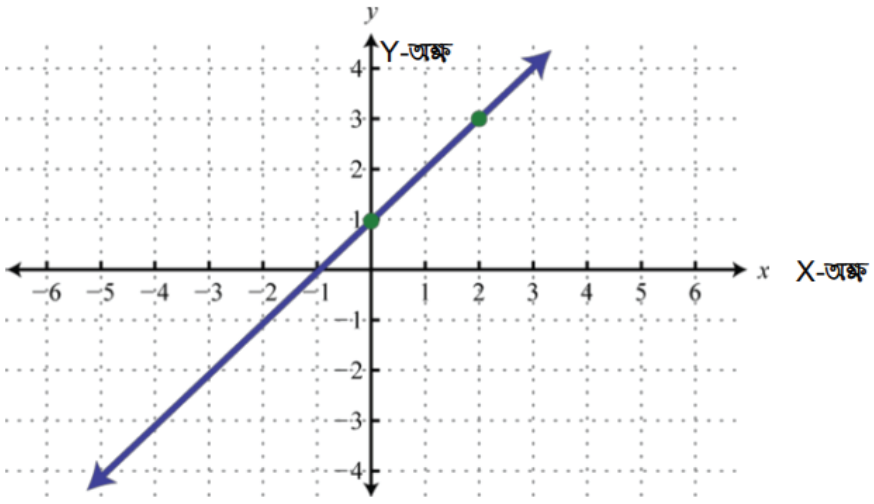
ডোমেন: স্বাধীন চলকের সকল গ্রাহ্য মানের সেট।

রেঞ্জ: অধীন চলকের সকল সম্ভাব্য মানের সেট।

(ক) এক-এক ফাংশন (One-One Function)
(খ) সার্বিক ফাংশন (Onto Function)

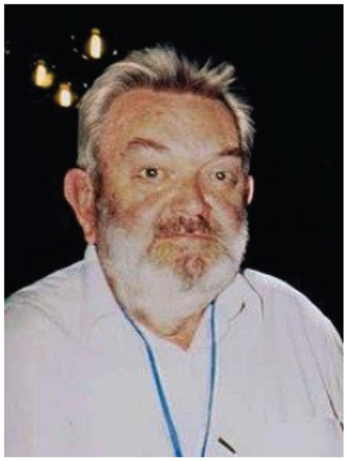
বিপরীত ফাংশন (Inverse Function): একটি ফাংশন যখন একইসাথে এক-এক এবং সার্বিক ফাংশন হয় তখন ঐ ফাংশনের বিপরীত ফাংশন বিদ্যমান থাকে। যেমন, f ফাংশনের বিপরীত ফাংশন f^{-1}.

ফাংশনের লেখচিত্র: লেখচিত্র একটি ফাংশনের জ্যামিতিক চিত্র প্রকাশ করে।



৩. কম্পিউটার অ্যালজেবরা সিস্টেম Computer Algebra System (CAS)

কম্পিউটার অ্যালজেবরা সিস্টেম যে কোনো একটি গাণিতিক সফ্টওয়্যার যা প্রচলিত গাণিতিক অভিব্যাক্তিকে প্রকাশ করতে সক্ষম। বিংশ শতকের দ্বিতীয়ার্ধে কম্পিউটার অ্যালজেবরা সিস্টেম এর বিকাশ ঘটে, যা গাণিতিক বস্তুর উপর অ্যালগরিদমের মাধ্যমে প্রয়োগ করা হয়। বিশেষত: গাণিতিক লেখচিত্র অংকনে কম্পিউটার অ্যালজেবরা সিস্টেম এর একটি উল্লেখযোগ্য ভূমিকা রয়েছে। কম্পিউটার অ্যালজেবরা সিস্টেমের ব্যবহার গাণিতিক লেখচিত্র অংকনকে সহজতর ও বাস্তবিক রুপ দান করেছে। ডাচ পদার্থবিজ্ঞানী **মার্টিনাস জ্যাস্টিনুস গডফ্রিডুস ভেল্টম্যান** কম্পিউটার অ্যালজেবরা সিস্টেম এর প্রবর্তক।



মার্টিনাস জ্যাস্টিনুস গডফ্রিডুস ভেল্টম্যান
(১৯৩১ – ২০২১)
নোবেল বিজয়ী ডাচ পদার্থবিজ্ঞানী
কম্পিউটার অ্যালজেবরা সিস্টেম এর পথিকৃত

GeoGebra একটি কম্পিউটার অ্যালজেবরা সিস্টেম, যা অস্ট্রিয়ান গণিতজ্ঞ **মার্কুস হোহেনওয়ার্টার** (অধ্যাপক, জোহানেস কেপলার ইউনিভার্সিটি, অস্ট্রিয়া) ২০০২ সালে প্রবর্তন করেছেন। অ্যালজেবরা, জ্যামিতি, এবং ত্রিকোণমিতিসহ গণিতের সকল শাখায় GeoGebra ব্যাপক জনপ্রিয়তা অর্জন করেছে।

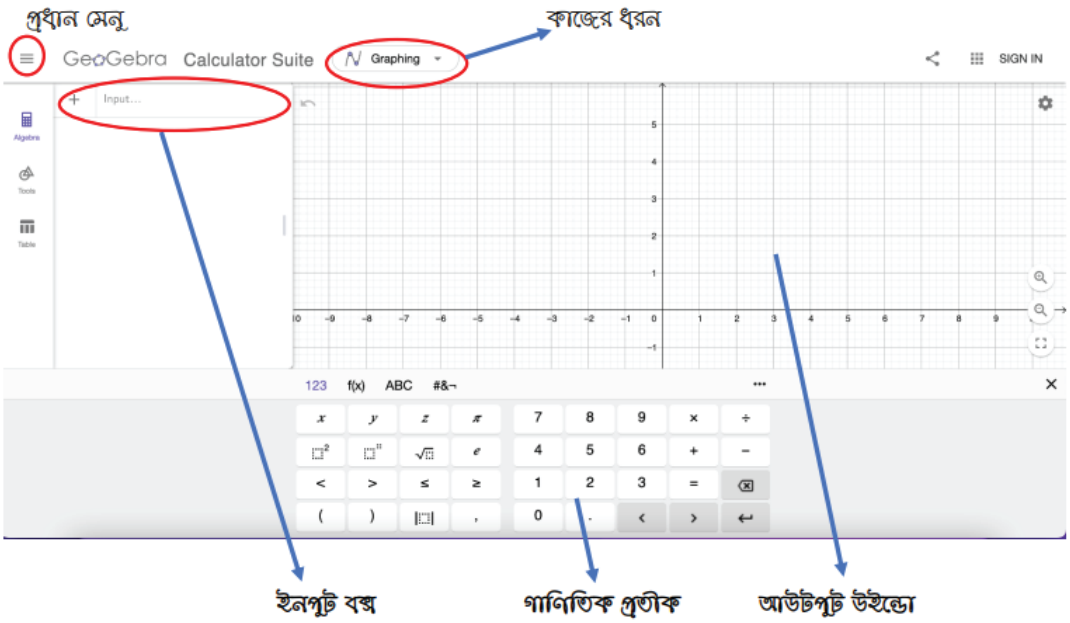
https://www.geogebra.org/calculator

GeoGebra

https://www.geogebra.org/calculator

জিওজেব্রা (GeoGebra) হল একটি ইন্টারেক্টিভ অ্যাপ্লিকেশন যা প্রাথমিক বিদ্যালয় থেকে বিশ্ববিদ্যালয় স্তর পর্যন্ত গণিত শেখার ও শেখানোর উদ্দেশ্যে তৈরী করা হয়েছে। এটি একটি ফ্রি ওপেনসোর্স সফ্টওয়্যার এবং উইন্ডোস, ম্যাকিনটস, লিনাক্স, এবং অ্যান্ড্রয়েড সিস্টেমে ব্যবহার উপযোগী। উপরন্তু, জিওজেব্রা অ্যাপ ইন্সটল না করে সরাসরি অনলাইনেও ব্যবহার করা সম্ভব।

জিওজেব্রা ইন্টারফেস (GeoGebra Interface)

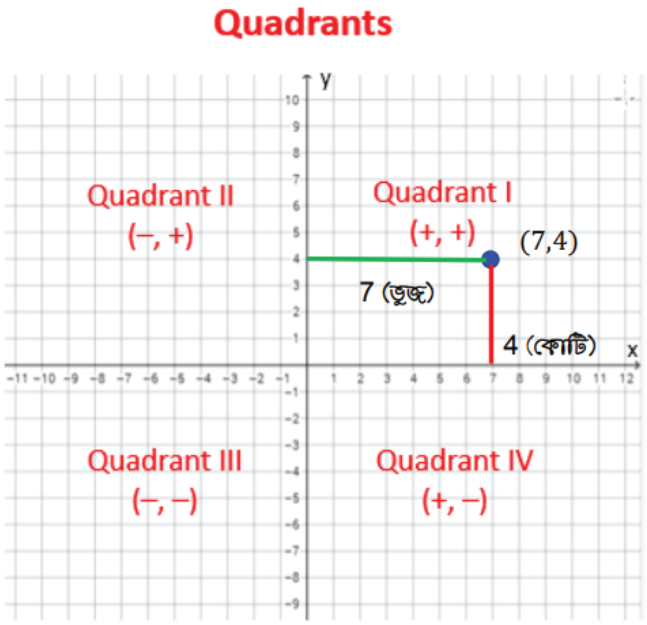


ড. মার্কুস হোহেনওয়ার্টার
অধ্যাপক
জোহানেস কেপলার ইউনিভার্সিটি, অস্ট্রিয়া
জিওজেব্রার (GeoGebra) উদ্ভাবক।

প্রশিক্ষণ কর্মশালা: *Teaching and Learning Mathematics using CAS at Secondary Level*

৪. স্থানাংক জ্যামিতি (Coordinate Geometry)

সপ্তদশ শতাব্দীতে ফরাসী গণিতজ্ঞ ও দার্শনিক **রেনে দেকার্ত (১৫৯৬–১৬৫০)** স্থানাংক জ্যামিতির ধারণা আবিষ্কার করেন, যা আয়তাকার কার্তেসীয় স্থানাংক (Rectangular Cartesian Coordinates) নামে পরিচিত। কার্তেসীয় স্থানাংকের মূলমন্ত্র হল যে কোনো বিন্দুর অবস্থান একটি আনুভূমিক (Horizontal) সরলরেখা এবং একটি উলম্ব (Vertical) সরলরেখা দ্বারা নির্ধারণ করা সম্ভব। পরস্পরছেদী এবং পরস্পর লম্ব দুইটি সরলরেখা দ্বারা আয়তাকার কার্তেসীয় স্থানাংক কাঠামো তৈরী হয়। ঐ সরলরেখা দুইটি পরস্পরছেদ করার ফলে চারটি চতুর্ভাগ (Quadrant) তৈরী হয় এবং ছেদবিন্দুকে মূলবিন্দু (Origin) নামে অভিহিত করা হয়। অধিকন্তু আনুভূমিক (Horizontal) সরলরেখাকে x অক্ষ এবং একটি উলম্ব (Vertical) সরলরেখাকে y অক্ষ দ্বারা সূচীত করা হয়। একটি বিন্দুর অবস্থান তার স্থানাংক দ্বারা নির্ধারিত হয়, স্থানাংক হল একটি ক্রমজোড়, যা ভুজ ও কোটির সমাবেশ, যেখানে ভুজ, y অক্ষ থেকে বিন্দুটির দূরত্ব এবং কোটি, x অক্ষ থেকে বিন্দুটির দূরত্ব নির্দেশ করে।



আয়তাকার কার্তেসীয় স্থানাংক

৫. সমীকরণের লেখচিত্র (Graph of Equations)

(ক) সরলরৈখিক সমীকরণ (Linear Equation): যে সমীকরণের সর্বোচ্চ ঘাত এক।

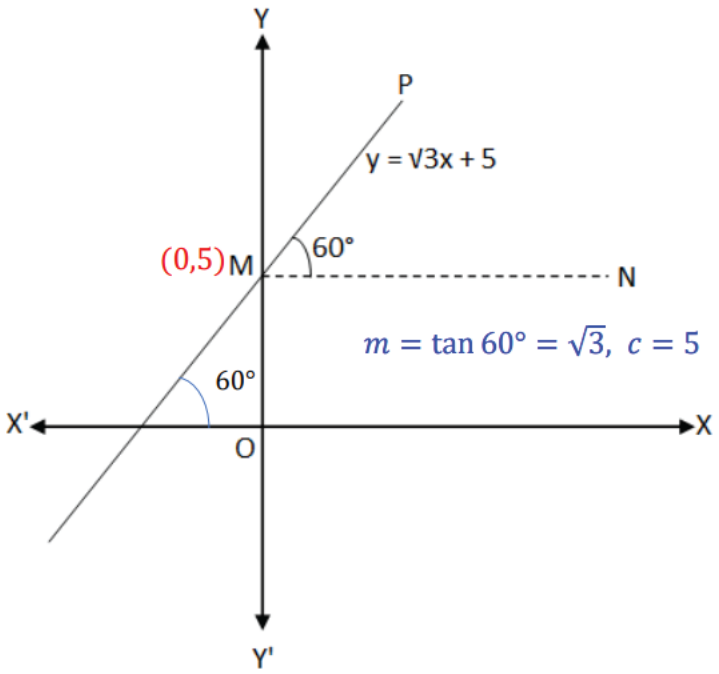
$$y = mx + c$$

এখানে m হল ঢাল (Slope) এবং c হল y- ইন্টারসেপ্ট (Intercept)।

ঢাল: ঢাল হল সরলরেখার উপর যেকোনো দুইটি বিন্দুর ভুজদ্বয়ের ব্যবধান এবং কোটিদ্বয়ের ব্যবধানের অনুপাত। অন্যদিকে সরলরেখাটি x অক্ষের ধনাত্মক দিকের সাথে যে কোণ উৎপন্ন করে তার ত্রিকোণমিতিক ট্যানজেন্ট (Tangent) এর মানকে ঐ সরলরেখার ঢাল বলে, অর্থাৎ $m = \tan\theta$।

y- ইন্টারসেপ্ট: y- ইন্টারসেপ্ট একটি বিন্দুর কোটি যেখানে সরলরেখাটি y অক্ষকে অতিক্রম করে।

ঢাল এবং y- ইন্টারসেপ্ট এর ভিন্নতার কারণে সরলরেখার প্রকৃতি ভিন্ন হয়।



সরলরৈখিক ফাংশনের লেখচিত্র

(খ) দ্বিঘাত সমীকরণ (Quadratic Equation): যে সমীকরণের সর্বোচ্চ ঘাত দুই।

$$ax^2 + bx + c = 0$$

এখানে a, b ও c যেকোনো বাস্তব সংখ্যা এবং $a \neq 0$।

দ্বিঘাত সমীকরণের মূলদ্বয়: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

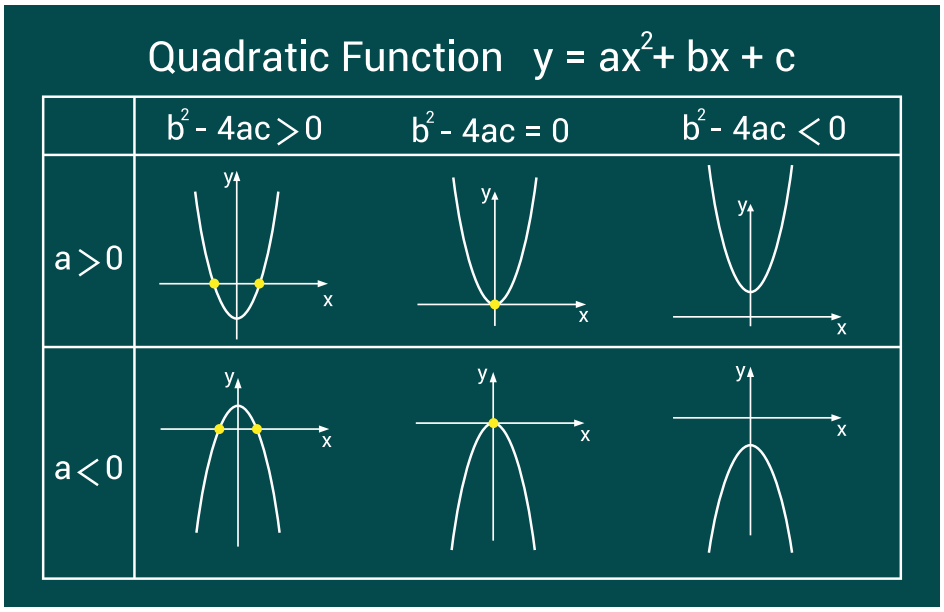
মূলদ্বয়ের প্রকৃতি সম্পূর্ণভাবে নিশ্চায়কের $b^2 - 4ac$ মানের উপর নির্ভরশীল।

নিশ্চায়কের মান	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
মূলদ্বয়ের প্রকৃতি	বাস্তব ও অসমান	বাস্তব ও সমান	অবাস্তব

দ্বিঘাত ফাংশনের $f(x) = ax^2 + bx + c$ লেখচিত্র:

দ্বিঘাত ফাংশনের লেখচিত্র, এর অগ্রণী সহগ a এর মান দ্বারা নিয়ন্ত্রিত হয়।

যখন $a > 0$ তখন দ্বিঘাত ফাংশনের লেখচিত্র উপরে অবতল এ $a < 0$ এর জন্য লেখচিত্রটি নিচে অবতল হয়।



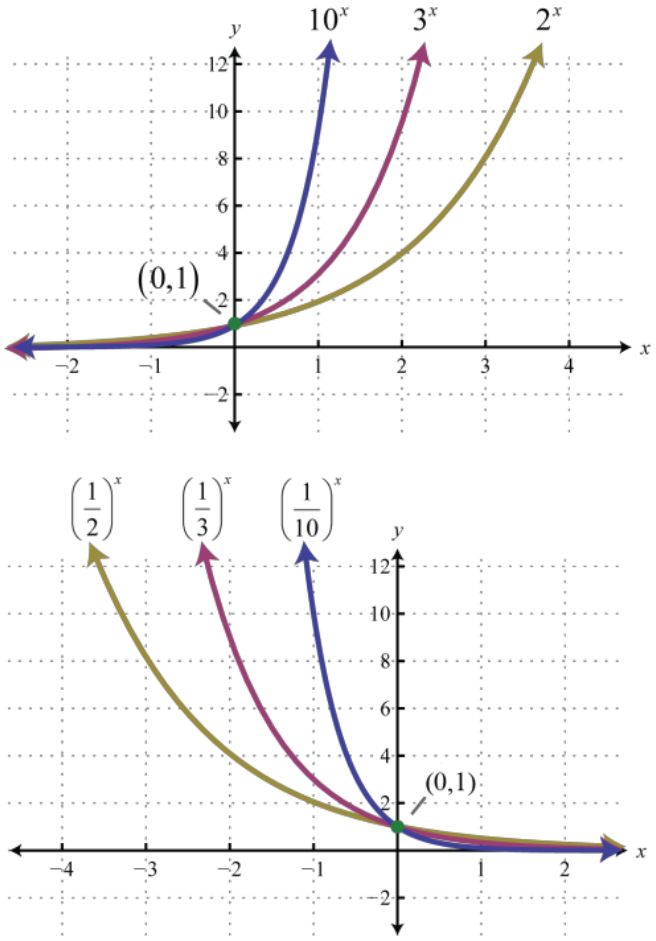
দ্বিঘাত ফাংশনের লেখচিত্র

৬. সূচকীয় ও লগারিদমীয় ফাংশনের লেখচিত্র (Graph of Exponential and Logarithmic Functions)

(ক) সূচকীয় ফাংশন (Exponential Function): সূচকীয় ফাংশনের সাধারণ রূপ: $f(x) = a^x$ যেখানে a একটি ধনাত্মক বাস্তব সংখ্যা যেখানে $a \neq 1$ এবং x যেকোনো বাস্তব সংখ্যা। এখানে, a কে ভিত্তি এবং x কে সূচক হিসাবে চিহ্নিত করা হয়।

ডোমেন: সকল বাস্তব সংখ্যার সেট, $(-\infty, \infty)$।

রেঞ্জ: সকল ধনাত্মক বাস্তব সংখ্যার সেট, $(0, \infty)$।

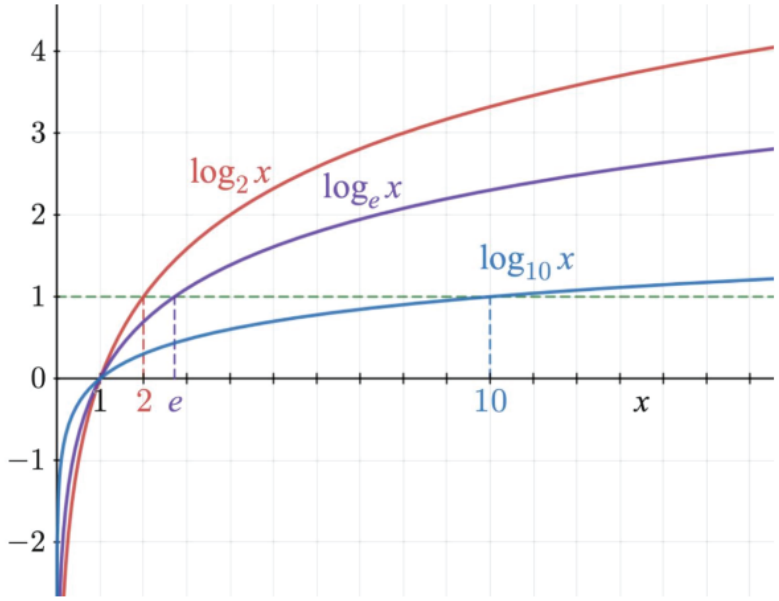


সূচকীয় ফাংশনের লেখচিত্র

(খ) লগারিদমীয় ফাংশন (Logarithmic Function): b ভিত্তিক লগারিদমীয় ফাংশনের সাধারণ রূপ: $f(x) = \log_b x$ যেখানে b একটি ধনাত্মক বাস্তব সংখ্যা কিন্তু $b \neq 1$ এবং x যেকোনো একটি ধনাত্মক বাস্তব সংখ্যা। এখানে $f(x)$ হলো x এর b ভিত্তিক লগারিদম।

ডোমেন: সকল ধনাত্মক বাস্তব সংখ্যার সেট, $(0, \infty)$।

রেঞ্জ: সকল বাস্তব সংখ্যার সেট, $(-\infty, \infty)$।



লগারিদমীয় ফাংশনের লেখচিত্র

সাধারণ লগারিদম (Common Logarithm): সাধারণ লগারিদম একটি ১০ ভিত্তিক লগারিদম, যেমন $\log_{10} x, \log_{10} y$। সাধারণ লগারিদম এর ক্ষেত্রে ভিত্তিটি উহ্য থাকে, যেমন $\log 2, \log 5$।

$$y = \log x \Leftrightarrow x = 10^y$$

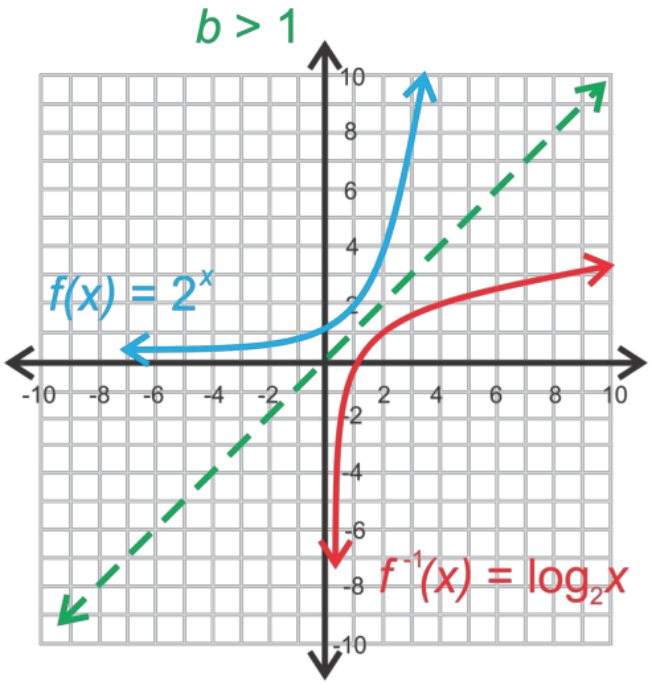
প্রাকৃতিক লগারিদম (Natural Logarithm): প্রাকৃতিক লগারিদম একটি e ভিত্তিক লগারিদম, যেমন $\log_e x, \log_e y$। প্রাকৃতিক লগারিদম এর ক্ষেত্রেও ভিত্তিটি উহ্য থাকে এবং log এর পরিবর্তে ln ব্যবহৃত হয়, যেমন $\ln 2, \ln 5$।

$$y = \ln x \Leftrightarrow x = e^y$$

(গ) সূচকীয় ফাংশন এবং লগারিদমীয় ফাংশন এর সম্পর্ক (Relation between Exponential and Logarithmic Functions):

$$y = a^x \Leftrightarrow x = \log_a y$$

ইনপুট এবং আউটপুট পরস্পর দ্বারা প্রতিস্থাপিত হয়েছে, যার ফলে সূচকীয় ফাংশন এবং লগারিদমীয় ফাংশন একে অপরের বিপরীত ফাংশন (Inverse Function) হিসাবে বিবেচিত হয়।



একে অপরের বিপরীত ফাংশন হিসাবে সূচকীয় ও লগারিদমীয় ফাংশনের লেখচিত্র

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